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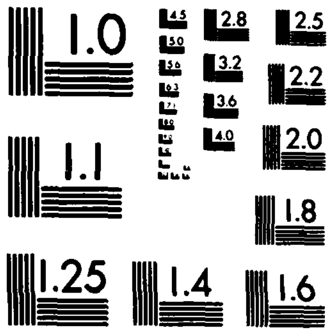

MICROCOPY RESOLUTION TEST CHART
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AD-A132 945

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

THE USE OF A MICROCOMPUTER SYSTEM AS AN AID
TO CLASSICAL AND DIGITAL CONTROL
SYSTEM DESIGN AND ANALYSIS

by

John Douglas Humphrey

June 1983

Thesis Advisor: Marle D. Hewett

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This thesis takes five FORTRAN IV programs from "Computer Programs for Computational Assistance in the Study of Linear Control Theory" by Melsa and Jones and translates them into a microcomputer BASIC language to run on an inexpensive microcomputer system. Three of the five programs are state variable programs. They are BASMAT for basic matric manipulation, RTRESP for rational time response, and GTRESP for graphical time response. Two are transfer function programs, FRESP for frequency response

20. ABSTRACT Continued

and RTLOC for root locus. A user's guide and example are included for each.

A final example is used to demonstrate the utility of the two transfer function programs as an aid to direct digital design in the w'-plane.

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The Use of a Microcomputer System as an Aid to Classical and Digital Control System Design and Analysis

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ABSTRACT

This thesis takes five FORTRAN IV programs from "Computer Programs for Computational Assistance in the Study of Linear Control Theory" ~~by Melsa and Jones~~ and translates them into a microcomputer BASIC language to run on an inexpensive microcomputer system. Three of the five programs are state variable programs. They are BASMAT for basic matrix manipulation, RTRESP for rational time response, and GTRESP for graphical time response. Two are transfer function programs, FRESP for frequency response and RTLOC for root locus. A user's guide and example are included for each.

A final example is used to demonstrate the utility of the two transfer function programs as an aid to direct digital design in the w'-plane.

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I. INTRODUCTION

The purpose of this thesis is two fold. First, it is to show that standard computer programs useful in the study of linear control theory may be adapted to run on an inexpensive home/microcomputer. To demonstrate this, five programs were chosen from [Ref. 1], three state variable programs and two transfer function programs. The three state variable programs are BASMAT, a basic matrix manipulation program, RTRESP, a rational time response program, and GTRESP, a graphical time response program. The two transfer function programs are FRESP, a frequency response program, and RTLOC, a root locus program. These programs as they appear in [Ref. 1] are written in basic FORTRAN IV language to be run on a main frame computer system utilizing standard graphics subroutines. These programs were modified and rewritten in a microcomputer BASIC language which is an interpreted language. Generally these programs are limited to systems of 10th order. It is felt that this limitation is more than acceptable for the purposes of this thesis. Also no major effort has been made to analyze or improve the efficiency of the numerical methods used. An effort of this type is advised if these programs are to be modified for higher order systems.

The second part of this thesis investigates the methods and relationships involved in direct digital design in the w'-plane. The two transfer function programs adapted to run on a microcomputer from the first part of this thesis, FRESP and RTLOC, are used to aid in this investigation.

Section II summarizes the common problems and considerations involved in the translation of the five programs to be run on a microcomputer system. Section III describes the three state variable programs, BASMAT, RTRESP, and GTRESP, and gives an example of their use and output. Section IV is similar to section III and describes the two transfer function programs, FRESP and RTLOC.

Section V deals with the w'-plane. Subsection A gives some background on the w'-plane and subsection B provides a simple example using the two transfer function programs to compare the s and w' planes. Subsection C develops templates of some constant parameters in the w'-plane. Subsection D ends the section with a more involved example.

Section VI provides some conclusions and recommendations.

II. TRANSLATION CONSIDERATIONS

The five programs in this thesis, BASMAT, RTRESP, GTRESP, FRESP, and RTLOC, were translated directly from programs of the same name in [Ref. 1]. These programs were written in the basic FORTRAN IV computer language, a compiled language designed to run on a main frame or mini computer system. In the first part of this thesis these programs are translated into a microcomputer BASIC language adapted to run on the microcomputer system described in Appendix A.

In general the translation from FORTRAN IV to the microcomputer BASIC used posed no serious problems. Even though the BASIC language used is by necessity an abridged version of the BASIC language found on larger more expensive systems, it was extensive enough to provide the necessary commands for these programs.

The input portion of the programs were rewritten to be interactive for convenient keyboard entry of problem parameters eliminating input formatting errors. The programs requiring more extensive input were given the added capability of saving and retrieving a problem description from a disk file for the user's convenience.

Unlike FORTRAN IV the BASIC used does not have complex math capabilities. Therefore, mathematical operations on complex quantities were programed separately for the real and imaginary parts.

Another important difference in the languages affecting translation is that BASIC has no provision for local variables. All variables in the main program and all subroutines are global. This caused some bookkeeping problems in translating subroutines to prevent undesired side effects. Another complication was that FORTRAN IV considers the first four characters of a variable name for identification of that unique variable. The BASIC used considers only the first two. This further complicated the bookkeeping of variables and resulted in variable names being assigned just because they were different from the rest and with no relation to the quantity represented.

The output routines were written to conform as close as possible to the FORTRAN IV version. Since the microcomputer system used is limited to eighty columns on eight and a half inch wide paper, provisions were made within the program to automatically switch to a condensed character font when necessary to output greater than eighty characters per line.

The programs requiring graphical output created some unique problems. First, no standard library subroutines for graphics as used in the FORTRAN IV programs were available on the microcomputer system used. As a result all graphics routines had to be developed for the system used. Also the appropriate variables were redimensioned giving consideration to the resolution of the graphics available on the microcomputer system to optimize the program somewhat. Secondly, although it is possible to mix graphics and text on the microcomputer system, it is not done in a straight forward manner and utilizes extra memory. For memory considerations and convenience it was decided to provide sufficient information for interpretation of the graphical output below each graph. It is felt that this method is satisfactory and creates little inconvenience to the user. To enhance the interpretation of the graphical output much time and effort was devoted to developing plotting routines to display the data in relation to axes labeled with tic-marks and boundaries of integer vice fractional values.

A major obstacle that was overcome was the identification and correction of a memory management problem unique to the microcomputer system used. This problem affected only those programs requiring graphical output. The essence of the problem was due to the size of the programs and the number of variables used, parts of the program and stored variables were being over written in memory by the graphics routines. This problem was finally solved by making appropriate changes in the memory management scheme. For a more detailed explanation of the solution see Appendix A.

III. STATE VARIABLE PROGRAMS

The three state variable programs discussed in this section are modified versions of the programs of the same names found in [Ref. 1]. These programs may be used as tools for the analysis and design of linear control systems expressed in the following state variable form:

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}\underset{\sim}{x}(t) + \underset{\sim}{b}u(t)$$

$$u(t) = K[r(t) - \underset{\sim}{k}^T\underset{\sim}{x}(t)]$$

$$y(t) = \underset{\sim}{c}^T\underset{\sim}{x}(t)$$

The Basic Matrix program (BASMAT) described in part A of this section is used to compute the determinant, inverse, characteristic polynomial, and eigenvalues of the square matrix $\underset{\sim}{A}$. It is also used to determine the resolvent matrix $(s\underset{\sim}{I}-\underset{\sim}{A})^{-1}$ and state transition matrix $\exp(\underset{\sim}{A}t)$.

The Rational Time Response program (RTRESP), described in part B, is used to determine a closed form expression for the time response of a system. The input function r(t) is required to have a rational time response and no repeated eigenvalues are allowed in the combination of the system and input.

The Graphical Time Response program (RTRESP), described in part C, is used to produce a graphical display of the time response of a system to an arbitrary input.

RTRESP and GTRESP can be used to study open loop systems by letting K equal zero and unforced systems by letting r(t) equal zero.

A. BASMAT

1. *Basic Matrix Program (BASMAT)*

a. Introduction

When done by hand, matrix manipulations can be quite tedious and the chances of an error being made are great. In the study of linear state variable analysis, a computer program to do these manipulations is almost essential. It can do the necessary manipulations much more quickly and accurately and allow the user to devote his time and energy to design and analysis.

b. Description of Program

BASMAT [Ref. 1: pp. 7,8] takes a matrix $\underset{\sim}{A}$, and computes the determinant of $\underset{\sim}{A}$ (det $\underset{\sim}{A}$), the inverse of $\underset{\sim}{A}$ ($\underset{\sim}{A}^{-1}$), the characteristic polynomial (det(s$\underset{\sim}{I}$-$\underset{\sim}{A}$)), and eigenvalues of $\underset{\sim}{A}$ (λ_i) as well as the resolvent matrix

$$\underset{\sim}{\Phi}(s) = (s\underset{\sim}{I}-\underset{\sim}{A})^{-1}$$

and the state transition matrix

$$\underset{\sim}{\Phi}(t) = \exp(\underset{\sim}{A}t)$$

The state transition matrix is expressed as matrix coefficients times the natural modes exp(λt) with the complex eigenvalues expressed as damped sine and cosine terms. The resolvent matrix is written as

$$\underset{\sim}{\Phi}(s) = \mathrm{adj}(s\underset{\sim}{I}-\underset{\sim}{A})/\det(s\underset{\sim}{I}-\underset{\sim}{A})$$

and the matrix numerator, adj(s$\underset{\sim}{I}$-$\underset{\sim}{A}$), is output as matrix coefficients of powers of s so that it takes the form

$$\mathrm{adj}(s\underset{\sim}{I}-\underset{\sim}{A}) = F_1 + F_2s + \ldots + F_Ns^{N-1}$$

The BASMAT program interactively accepts input of a matrix and calls subroutines to perform the appropriate calculations. The subroutines used are CHREQ, CHREQA, PROOT, DET, SIMEQ, and STMST. These subroutines are listed below with a brief description.

CHREQ. This subroutine is used to determine the characteristic polynomial $\det(s\underset{\sim}{I}-\underset{\sim}{A})$, and the resolvent matrix $(s\underset{\sim}{I}-\underset{\sim}{A})^{-1}$ for the matrix $\underset{\sim}{A}$. The Leverrier algorithm is used to compute the resolvent matrix

$$\underset{\sim}{\phi}(s) = (s\underset{\sim}{I}-\underset{\sim}{A})^{-1}$$

in the form

$$\underset{\sim}{\phi}(s) = \frac{\sum_{i=1}^{N} R_i s^{N-i}}{D(s)}$$

where $D(s)$ is the characteristic polynomial $\det(s\underset{\sim}{I}-\underset{\sim}{A})$.

The coefficients of the characteristic polynomial are determined by subroutine CHREQA.

CHREQA. This subroutine is called by CHREQ to determine the characteristic polynomial, $\det(s\underset{\sim}{I}-\underset{\sim}{A})$, for the $\underset{\sim}{A}$ matrix. The principal-minor method is used.

PROOT. This subroutine uses a modified Bairstow method for root extraction to determine the roots of a polynomial with real coefficients. It is used to calculate the eigenvalues of the $\underset{\sim}{A}$ matrix.

DET. This subroutine computes the determinant of a matrix. A gauss elimination method to place the matrix in upper triangular form is used. It is called by CHREQA to calculate sub-determinants.

SIMEQ. This subroutine is used in the FORTRAN version of BASMAT in [Ref. 1: pp. 7,8] to determine the inverse of the $\underset{\sim}{A}$ matrix. In the BASIC version a machine language subroutine [Ref. 2] is substituted for speed and convenience.

STMST. This subroutine is used to compute the state transition matrix

$$\Phi(t) = \exp(\underset{\sim}{A}t)$$

for a matrix $\underset{\sim}{A}$. It uses the Sylvester Expansion Theorem. Eigenvalues of the $\underset{\sim}{A}$ matrix must be provided. This routine can not handle duplicate eigenvalues.

c. Program Translation Problems

See section II, Translation Considerations.

2. BASMAT User's Guide

This program occupies about 7k bytes of memory and does not utilize the graphics pages, therefore it is not necessary to relocate the disk operating system.

Push "return" after each input.

STEP 1:

SCREEN PROMPT:

BASIC MATRIX PROGRAM

PROBLEM IDENTIFICATION ->

REMARKS:

Label the problem. Type in any appropriate combination of letters, numbers, and/or symbols, excluding commas. This input is limited to 255 characters.

STEP 2:

SCREEN PROMPT:

PLANT ORDER? ->

REMARKS:

Enter the number that corresponds to the order of the $\underset{\sim}{A}$ matrix. Maximum order allowed is ten.

STEP 3:

SCREEN PROMPT:

INPUT PLANT MATRIX.

A(row,column)=

REMARKS:

Input the $\underset{\sim}{A}$ matrix by typing in each element as prompted. The program will ask for each element beginning with the first row and going from left to right.

STEP 4:

SCREEN PROMPT:

HARDCOPY? (Y/N) ->

REMARKS:

After this step is completed the program will output the $\underset{\sim}{A}$ matrix for reference. This will be followed by the inverse of the $\underset{\sim}{A}$ matrix, the determinant of the $\underset{\sim}{A}$

matrix, the matrix coefficients of the resolvent matrix numerator, the characteristic polynomial coefficients, the eigenvalues of the plant matrix, and finally the elements of the state transition matrix. See Figure 1 for a sample output.

3. BASMAT Example

This example will use the system matrix

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -3 & -3 \end{bmatrix}$$

to demonstrate the use of the BASMAT program. It will refer to the steps described in the previous section, BASMAT User's Guide.

Step 1. Enter "EXAMPLE <CR>" where <CR> = return.

Step 2. Enter "3 <CR>"

Step 3. Enter "0 <CR> 1 <CR> 0 <CR> 0 <CR> 0 <CR> 1 <CR> -2 <CR> -3 <CR> -3 <CR>"

Step 4. Enter "Y <CR>"

The resulting output is shown in Figure 1. These results are interpreted as follows:

The plant matrix is shown just as it was entered with rows horizontal and columns vertical. This is the format used for all matrix output.

The inverse of the plant matrix is shown followed by the scalar determinant of the $\underset{\sim}{A}$ matrix.

The matrix coefficients of the resolvent matrix numerator are given as powers of s. The characteristic polynomial is listed as coefficients of powers of s.

From this output, the resolvent matrix

$$\underset{\sim}{\phi}(s) = \text{adj}(s\underset{\sim}{I}-\underset{\sim}{A})/\det(s\underset{\sim}{I}-\underset{\sim}{A})$$

may be written. The characteristic polynomial, $\det(s\underset{\sim}{I}-\underset{\sim}{A})$, is $2+3s+3s^2+s^3$. The first element of the resolvent matrix numerator is $3+3s+s^2$ making the first element of the resolvent matrix

$$\phi_{11}(s) = \frac{3+3s+s^2}{2+3s+3s^2+s^3}$$

The real and imaginary parts of the eigenvalues of the plant matrix are listed. Finally, the elements of the state transition matrix are given. The first element of the state transition matrix can be written

$$\phi_{11}(t) = 0.333e^{-2t}+0.667e^{-0.5t}\cos 0.866t+1.15e^{-0.5t}\sin 0.866t$$

```
BASIC MATRIX PROGRAM
PROBLEM IDENTIFICATION-EXAMPLE

********************************

THE PLANT MATRIX

0             1             0
0             0             1
-2            -3            -3

********************************

THE INVERSE OF THE PLANT MATRIX

-1.5          -1.5          -.5
1             0             0
-0            1             0

********************************


THE DETERMINANT OF THE PLANT MATRIX

-2


********************************

THE MATRIX COEFFICIENTS OF THE RESOLVENT MATRIX NUMERATOR


THE MATRIX COEFFICIENT OF S^2

1             0             0
0             1             0
0             0             1

THE MATRIX COEFFICIENT OF S^1

3             1             0
0             3             1
-2            -3            0

THE MATRIX COEFFICIENT OF S^0

3             3             1
-2            0             0
0             -2            0

********************************
```

Figure 1 BASMAT Output

Figure 1 (Cont.)

THE CHARACTERISTIC POLYNOMIAL IN ASCENDING POWERS

2 3 3 1

****

THE EIGENVALUES OF THE PLANT MATRIX

REAL	IMAGINARY
-2	0
-.5	-.866025404
-.5	.866025404

****

THE ELEMENTS OF THE STATE TRANSITION MATRIX

THE MATRIX COEFFICIENT OF EXP(-2)T

.333333333	.333333333	.333333333
-.666666666	-.666666666	-.666666666
1.33333333	1.33333333	1.33333333

THE MATRIX COEFFICIENT OF EXP(-.5)T*COS(.866025404)T

.666666666	-.333333333	-.333333333
.666666666	1.66666667	.666666666
-1.33333333	-1.33333333	-.333333333

THE MATRIX COEFFICIENT OF EXP(-.5)T*SIN(.866025404)T

1.15470054	1.73205081	.577350268
-1.15470054	-.577350268	0
0	-1.15470054	-.577350268

****

Figure 1

B. RTRESP

1. Rational Time Response Program (RTRESP)

a. Introduction

Frequently it is desirable to know the response of a system as a function of time. A computer program can determine this quicker and more accurately than by hand.

b. Description of Program

RTRESP determines the time response in closed form of the closed loop system

$$\dot{\underset{\sim}{x}}(t)=\underset{\sim}{A}\underset{\sim}{x}(t)+\underset{\sim}{b}u(t)$$

$$u(t)=K[r(t)-\underset{\sim}{k}^T\underset{\sim}{x}(t)]$$

$$y(t)=\underset{\sim}{c}^T\underset{\sim}{x}(t)$$

due to any initial conditions $\underset{\sim}{x}(0)$ and input r(t) for $t\geq 0$. The system must have a rational Laplace transform R(s) with a pole-zero excess of at least one. [Ref. 1: pp. 11,12]

The input r(t) is treated by forming a m^{th}-order dynamic system whose initial condition response is equal to r(t) for a specific set of initial conditions. This system is combined with the original system and then the complete response in closed form is determined from the subroutine STMST. The order of the combined system must be ten or less.

Various primary and utility subroutines are used in RTRESP. The primary subroutines are listed below with a brief description:

CHREQA. This subroutine determines the characteristic polynomial, det($s\underset{\sim}{I}-\underset{\sim}{A}$), for the matrix $\underset{\sim}{A}$ using the principal-minor method.

PROOT. This subroutine uses a modified Bairstow method for root extraction to determine the roots of a polynomial with real coefficients. It is used to determine the roots of the numerator and denominator of R(s) when they are entered in polynomial form. PROOT is also used to determine the eigenvalues of the combined system matrix (i.e. the roots of the characteristic polynomial determined by CHREQA).

SEMBL. This subroutine determines the coefficients of a polynomial from the roots of the polynomial. It is used to determine the polynomial coefficients of the numerator and denominator of R(s) when they are entered in the factored form. This subroutine together with PROOT provide the feature that R(s) may be entered in either factored or polynomial form. It will appear in the output in both forms.

STMST. This subroutine computes the state transition matrix

$$\underset{\sim}{\phi}(t) = \exp(\underset{\sim}{A}t)$$

for a matrix $\underset{\sim}{A}$. It uses the Sylvester Expansion Theorem. Eigenvalues of the $\underset{\sim}{A}$ matrix must be provided. This routine can not handle duplicate eigenvalues, therefore it is necessary that the combined system and input have no repeated eigenvalues. STMST is used to determine the state transition matrix of the augmented system.

c. Program Translation Problems

See section II, Translation Considerations.

2. <u>RTRESP User's Guide</u>

Push "return" after each input.

STEP 1:

SCREEN PROMPT:

```
RATIONAL TIME RESPONSE PROGRAM (RTRESP)

THIS PROGRAM DETERMINES THE TIME RESPONSE OF A
CLOSED-LOOP SYSTEM DUE TO SPECIFIED INITIAL
CONDITIONS AND INPUT.  SYSTEM MUST HAVE A RA-
TIONAL LAPLACE TRANSFORM WITH A POLE-ZERO EXCESS
OF AT LEAST ONE.

PROBLEM IDENTIFICATION ->
```

REMARKS:

Label the problem. Type in any appropriate combination of letters, numbers, and/or symbols, excluding commas and colons. This input is limited to 255 characters. This is also the name used to save and retrieve the disk file containing the problem description. (See steps 2 and 3)

STEP 2:

SCREEN PROMPT:

```
WAS THIS PROBLEM DESCRIPTION PREVIOUSLY
SAVED? (Y/N) ->
```

REMARKS:

This step provides the option of recalling a previously saved problem description from the disk.

If "Y" is typed, the program looks for the problem description saved under the label typed in STEP 1. The program then runs to completion using the retrieved program description. If an "N" is typed, the program continues with STEP 3.

STEP 3:

SCREEN PROMPT:

DO YOU WANT TO SAVE THIS PROBLEM DESCRIPTION?
(Y/N) ->

REMARKS:

A positive response will save the problem description, that will be entered in the following steps, to the disk under the label entered in STEP 1. This problem may be retrieved during a later session by entering the proper label in STEP 1 and typing "Y" in STEP 2.

STEP 4:

SCREEN PROMPT:

ORDER OF THE SYSTEM? ->

REMARKS:

The total order of the system and the input must not exceed ten.

STEP 5:

SCREEN PROMPT:

INPUT SYSTEM (A) MATRIX

A(1,1) =

REMARKS:

Enter the elements of the $\underset{\sim}{A}$ matrix as prompted.

The format is A(row,column) = .

STEP 6:

SCREEN PROMPT:

THE A MATRIX

(display of the $\underset{\sim}{A}$ matrix)

ANY CHANGES? (Y/N) ->

REMARKS:

If an "N" is typed, the program proceeds to STEP 9.

To correct the matrix type "Y" and the program will proceed to STEP 7.

STEP 7:

SCREEN PROMPT:

TYPE ROW,COLUMN OF THE ELEMENT TO BE
CORRECTED ->

STEP 8:

SCREEN PROMPT:

A (row,column) =

REMARKS:

Enter the correct value. The program will return to STEP 6.

STEP 9:

SCREEN PROMPT:

INPUT THE CONTROL (B) VECTOR

B(1) =

REMARKS:

After the $\underset{\sim}{B}$ vector is entered, a correction sequence similar to steps 6 through 8 is encountered.

STEP 10:

SCREEN PROMPT:

INPUT THE OUTPUT (C) VECTOR

C(1) =

REMARKS:

After the $\underset{\sim}{C}$ vector is entered, a correction sequence similar to steps 6 through 8 is encountered.

STEP 11:

SCREEN PROMPT:

INPUT THE FEEDBACK COEFFICIENTS

FEEDBACK COEFFICIENT (1) =

REMARKS:

After the feedback coefficients are entered, a correction sequence similar to steps 6 through 8 is encountered.

STEP 12:

SCREEN PROMPT:

INPUT THE GAIN ->

REMARKS:

Enter the controller gain.

STEP 13:

SCREEN PROMPT:

```
INPUT THE INITIAL CONDITIONS

XO(1) =
```

REMARKS:

After the initial conditions are entered, a correction sequence similar to steps 6 through 8 is encountered.

STEP 14:

SCREEN PROMPT:

```
ENTER THE INPUT GAIN ->
```

REMARKS:

Enter the gain of the input function, R(s).

STEP 15:

SCREEN PROMPT:

```
'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR NUMERATOR POLYNOMIAL ->
```

REMARKS:

Choose the preferred method of entering the numerator polynomial of the input function, R(s).

STEP 16A:

SCREEN PROMPT (assumes coefficient form chosen):

```
INPUT POLYNOMIAL COEFFICIENTS

INPUT COEFFICIENTS IN ASCENDING PWRS OF S

INPUT COEFFICIENT OF S^0 ->
```

REMARKS:

Enter the coefficient along with its algebraic sign as prompted. The program assumes that the coefficient of the highest power of s is one.

STEP 16B (assumes factored form chosen):

SCREEN PROMPT:

```
INPUT POLYNOMIAL FACTORS

FOR FACTOR 1
REAL PART ->

IMAGINARY PART ->
```

REMARKS:

Enter the factor with its algebraic sign. Do not enter the associated root. (e.g. For "(s-1)" enter "-1".) After the real part of the factor is entered, the program asks for the imaginary part, allowing for input of quadradic factors. If a non-zero imaginary part is entered, the program automatically enters its complex conjugate.

STEP 17:

SCREEN PROMPT:

```
'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR DENOMINATOR POLYNOMIAL ->
```

REMARKS:

The denominator is entered just as described for the numerator in steps 15 and 16.

After this step is completed, the program will output all input for reference followed by the time response of $\underset{\sim}{x}(t)$ and the system output, y(t).

3. RTRESP Example

It is desired to know the time response of the closed loop system

$$\dot{\underset{\sim}{x}}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \underset{\sim}{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$u(t) = 5\{r(t) - [1.0 \quad 0.5] \underset{\sim}{x}(t)\}$$

$$y(t) = [1 \quad 1] \underset{\sim}{x}(t)$$

if the input function is given by

$$R(s) = 2.0 \ (1/s)$$

and the initial conditions are

$$\underset{\sim}{x}(t) = \underset{\sim}{0}$$

Referring to the steps described in the pervious section, RTRESP User's Guide, this problem would be entered as follows:

Step 1. Enter "EXAMPLE <CR>" where <CR> = carriage return.

Step 2 and Step 3. Enter "N <CR>."

Step 4. Enter "2 <CR>."

Step 5. Enter "0 <CR> 1 <CR> -1 <CR> -1 <CR>."

Step 6. Enter "N <CR>."

Step 9. Enter "0 <CR> 1 <CR>." Enter appropriate response for correction sequence.

Step 10. Enter "1 <CR> 1 <CR>." Enter appropriate response for correction sequence.

Step 11. Enter "1 <CR> .5 <CR>." Enter appropriate response for correction sequence.

Step 12. Enter "5 <CR>."

Step 13. Enter "0 <CR> 0 <CR>." Enter appropriate response for correction sequence.

Step 14. Enter "2 <CR>."

Step 15. Enter "P,0 <CR>."

Step 16A. Enter "1 <CR>."

Step 17. Enter "F,1 <CR> 0 <CR>."

The program will now run to completion resulting in the output shown in Figure 2.

The output begins with the program name and problem identification. The $\underset{\sim}{A}$, $\underset{\sim}{B}$, and $\underset{\sim}{C}$ matrices are shown for reference followed by the feedback coefficients and the controller gain. Next, the initial conditions and input gain are listed. The numerator and denominator polynomials of the input function are given in both polynomial coefficient form and factored form regardless of the method in which they were entered. The polynomial coefficient form is given as a list of coefficients from left to right in ascending powers of s. The highest power of s is always one. The polynomial factored form appears as a list of the real and imaginary parts of each root of the polynomial. Notice that the numerator is only given in coefficient form since it is of zero order and no roots exist.

The time response of the state $\underset{\sim}{x}(t)$ is given in the form of vector coefficients of the various natural modes of the system and the input function. From this output $\underset{\sim}{x}(t)$ is seen to be

$$x_1(t) = -1.67e^{-1.75t} \cos 1.71t - 1.70e^{-1.75t} \sin 1.71t + 1.67$$

$$x_2(t) = 5.83e^{-1.75t} \sin 1.71t$$

The time response of the output y(t) is given as scalor coefficients of the same natural modes as $\underset{\sim}{x}(t)$ so it is seen that

$$y(t) = -1.67e^{-1.75t} \cos 1.71t + 4.13e^{-1.75t} \sin 1.71t + 1.67$$

```
RATIONAL TIME RESPONSE
PROBLEM IDENTIFICATION -> EXAMPLE
*************************************************

THE A MATRIX

0       1
-1      -1

THE B MATRIX

0       1
THE C MATRIX

1       1

FEEDBACK COEFFICIENTS

1       .5

GAIN = 5

*************************************************

INITIAL CONDITIONS, X(0)

0       0

RGAIN = 2




NUMERATOR POLYNOMIAL OF R(S) - ASCENDING PWRS OF S

1
```

Figure 2 RTRESP Output

Figure 2 (Cont.)

```
DENOMINATOR POLYNOMIAL OF R(S) - ASCENDING PWRS OF S

0      1

DENOMINATOR ROOTS ARE
REAL PART         IMAGINARY PART

0                 0

*************************************************

THE TIME RESPONSE OF THE STATE X(T)


THE VECTOR COEFFICIENT OF EXP(-1.75)T*COS(1.71391365)T
-1.66666667       0

THE VECTOR COEFFICIENT OF EXP(-1.75)T*SIN(1.71391365)T
-1.70175824       5.83459966

THE VECTOR COEFFICIENT OF EXP(0)T
1.66666667        9.31322575E-10

*************************************************

THE TIME RESPONSE OF THE OUTPUT Y(T)

THE COEFFICIENT OF EXP(-1.75)T*COS(1.71391365)T

-1.66666667
THE COEFFICIENT OF EXP(-1.75)T*SIN(1.71391365)T

4.13284143

THE COEFFICIENT OF EXP(0)T

1.66666667

*************************************************
```

Figure 2

C. GTRESP

1. Graphical Time Response Program (GTRESP)

a. Introduction

Knowing the response of a system as a function of time can be very helpful in the study and analysis of that system. The graphical display of this response can give much insight into a system's response characteristics.

b. Description of Program

GTRESP [Ref. 1: pp. 22-28] determines and graphically displays the time response of the closed loop system

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}\underset{\sim}{x}(t) + \underset{\sim}{b}u(t)$$

$$u(t) = K\,[r(t) - k^T \underset{\sim}{x}(t)]$$

$$y(t) = \underset{\sim}{c}^T \underset{\sim}{x}(t)$$

due to any initial conditions $\underset{\sim}{x}(0)$ and input r(t).

The basic purpose of this program is similar to that of the RTRESP program described earlier. The difference between the two programs is that GTRESP will determine the time response for arbitrary input functions which may not have rational Laplace transforms but RTRESP requires a rational Laplace transform. Also GTRESP produces a graphical display of the time response instead of a closed form expression as in the RTRESP program.

The GTRESP program uses a fourth-order Runge-Kutta numerical integration algorithm to calculate the time response.

The main subroutines used are CALCU, RUNGE, TRESP, and YDOT. Also various utility and plotting subroutines are used.

CALCU. This subroutine is called by TRESP to determine the reference input r(t) and the control input

$$u(t)=K\,[r(t) - \underset{\sim}{k}^T x(t)]$$

The reference input r(t) must be defined by the user by inserting the appropriate BASIC coding into the subroutine between line numbers 5010 and 5500. The reference input is represented by the variable R and the control input by the variable U.

RUNGE. This subroutine is called by TRESP and contains the actual fourth-order Runge-Kutta integration algorithm. It must be executed four times for each integration step.

TRESP. This subroutine is the driving subroutine which calls the subroutines CALCU, RUNGE, and YDOT along with the necessary plotting routines. It calculates the time response of the closed loop linear system described by the input parameters and plots the desired variables.

YDOT. This subroutine is called by TRESP to compute the derivative

$$\dot{\underset{\sim}{x}}(t)=\underset{\sim}{A}\underset{\sim}{x}(t)+\underset{\sim}{b}u(t)$$

It is designed to handle linear systems but can easily be modified to handle nonlinear and time varying systems which would give GTRESP a nonlinear and time varying capability.

c. Program Translation Problems

See section II, Translation Considerations.

2. GTRESP User's Guide

Push "return" after each input.

STEP 1:

SCREEN PROMPT:

GRAPHICAL TIME RESPONSE PROGRAM (GTRESP)

THIS PROGRAM DETERMINES AND GRAPHICALLY DISPLAYS THE TIME RESPONSE OF A CLOSED-LOOP SYSTEM DUE TO SPECIFIED INITIAL CONDITIONS AND INPUT.

PROBLEM IDENTIFICATION ->

REMARKS:

Label the problem. Type in any appropriate combination of letters, numbers, and/or symbols, excluding commas and colons. This input is limited to 255 characters. This is also the name used to save and retrieve the disk file containing the problem description. (See steps 2 and 3)

STEP 2:

SCREEN PROMPT:

WAS THIS PROBLEM DESCRIPTION PREVIOUSLY SAVED? (Y/N) ->

REMARKS:

This step provides the option of recalling a previously saved problem description from the disk. If "Y" is typed the program looks for the problem description saved under the label typed in STEP 1.

The program then runs to completion using the retrieved program description. If an "N" is typed the program continues with STEP 3.

STEP 3:

SCREEN PROMPT:

```
DO YOU WANT TO SAVE THIS PROBLEM DESCRIPTION?
(Y/N) ->
```

REMARKS:

The problem description will be entered in the following steps.

A positive response will save the problem description to the disk under the label entered in STEP 1. This problem may be retrieved during a later session by entering the proper label in STEP 1 and typing "Y" in STEP 2.

STEP 4:

SCREEN PROMPT:

```
ORDER OF THE SYSTEM? ->
```

REMARKS:

The total order of the system and the input must not exceed ten.

STEP 5:

SCREEN PROMPT:

```
INPUT SYSTEM (A) MATRIX
A(1,1) =
```

REMARKS:

Enter the appropriate element of the A matrix. The format is A(row,column) = .

STEP 6:

SCREEN PROMPT:

THE A MATRIX

(display of the A matrix)

ANY CHANGES? (Y/N) ->

REMARKS:

If an "N" is typed, the program proceeds to STEP 9. To correct the matrix type "Y" and the program will proceed to STEP 7.

STEP 7:

SCREEN PROMPT:

TYPE ROW,COLUMN OF THE ELEMENT TO BE
CORRECTED ->

STEP 8:

SCREEN PROMPT:

A(row,column) =

REMARKS:

Enter the correct value. The program will return to STEP 6.

STEP 9:

SCREEN PROMPT:

INPUT THE CONTROL (B) VECTOR

B(1) =

REMARKS:

After the $\underset{\sim}{B}$ vector is entered a correction sequence similar to steps 6 through 8 is encountered.

STEP 10:

SCREEN PROMPT:

INPUT THE OUTPUT (C) VECTOR

C(1) =

REMARKS:

After the $\underset{\sim}{C}$ vector is entered a correction sequence similar to steps 6 through 8 is encountered.

STEP 11:

SCREEN PROMPT:

INPUT THE FEEDBACK COEFFICIENTS

FEEDBACK COEFFICIENT (1) =

REMARKS:

After the feedback coefficients are entered a correction sequence similar to steps 6 through 8 is encountered.

STEP 12:

SCREEN PROMPT:

INPUT THE CONTROLLER GAIN ->

REMARKS:

Enter the controller gain.

STEP 13:

SCREEN PROMPT:

```
INPUT THE INITIAL CONDITIONS

XO(1) =
```

REMARKS:

After the initial conditions are entered a correction sequence similar to steps 6 through 8 is encountered.

STEP 14:

SCREEN PROMPT:

```
ENTER THE FOLLOWING PARAMETERS:
NOTE..(TF-TZ)/(DT*FR) =< 100

INITIAL TIME (TZ) ->
```

REMARKS:

Enter the initial time of the time interval of interest. Due to program constraints the initial and final times and the time step and frequency of output must be chosen so that they satisfy the relation.

(TF-TZ) / (DT*FR) =< 100

STEP 15:

SCREEN PROMPT:

```
FINAL TIME (TF) ->
```

REMARKS:

Enter the final time of the time interval of interest.

STEP 16:

SCREEN PROMPT:

TIME STEP (DT) ->

REMARKS:

Enter the time increment for each step.

STEP 17:

SCREEN PROMPT:

FREQUENCY OF OUTPUT (FR) ->

REMARKS:

Enter an integer n and the program will print out data on every nth time step iteration. Ensure that the relation expressed in the remarks of STEP 14 is satisfied.

STEP 18:

SCREEN PROMPT:

YOU MAY PLOT UP TO 8 VARIABLES VS TIME.

VARIABLE	NUMBER	VARIABLE	NUMBER
X1(T)	1	X8(T)	8
X2(T)	2	X9(T)	9
X3(T)	3	X10(T)	10
X4(T)	4	E(T)	11
X5(T)	5	U(T)	12
X6(T)	6	Y(T)	13
X7(T)	7	R(T)	14

HOW MANY VARIABLES TO PLOT? MAX=8 ->

REMARKS:

You may plot up to 8 variables vs time. These variables will be plotted on the same plot in the output. Enter the number of variables you want to appear on the plot.

STEP 19:

SCREEN PROMPT:

```
TYPE THE VARIABLE NUMBER <CR> ->
```

REMARKS:

Type the number associated with the variable of interest. After carriage return <CR> is typed, the program will continue to prompt for another variable number until the number of variables that the user indicated in STEP 18 has been entered.

STEP 20:

SCREEN PROMPT:

```
YOU CHOSE THE FOLLOWING VARIABLES:
(list of variable numbers entered in STEP 19)
DO YOU WANT TO MAKE ANY CHANGES? (Y/N) ->
```

REMARKS:

To make a change in the variables to be plotted type a "Y" and the program will return to STEP 18. If an "N" is typed, the program will run to completion.

3. GTRESP Example

It is desired to determine and plot the time response of the error

$$e(t)=r(t)-y(t)$$

the input r(t), and the state variable $x_2(t)$ for the system

$$\dot{\underset{\sim}{x}}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \underset{\sim}{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$u(t) = 5\{r(t) - [1.0 \quad 0.5]\ \underset{\sim}{x}(t)\}$$

$$y(t) = [1 \quad 1]\ \underset{\sim}{x}(t)$$

The time interval of interest is

$$0 \le t \le 4$$

The iteration step size (DT) is chosen to be 0.01 and printed output is desired every ten steps so FREQ equals 10. The initial conditions are

$$\underset{\sim}{x}(t)=0$$

and the input function is

$$r(t) = 5.0 \qquad \text{if } 0 \le t \le 1$$

$$r(t) = 0 \qquad \text{otherwise.}$$

In order to define this input function the following BASIC coding is inserted between line numbers 5010 and 5500 in the subroutine CALC.

```
5020 IF TZ > 1 GOTO 5040
5030 R = 5:  GOTO 5500
5040 R = 0
5500 REM  END OF ROUTINE DESCRIBING R(T)
```

Referring to the steps described in section 2, GTRESP User's Guide, this problem would be entered as follows:

Step 1. Enter "EXAMPLE <CR>" where <CR> = carriage return.

Step 2 and Step 3. Enter "N <CR>."

Step 4. Enter "2 <CR>."

Step 5. Enter "0 <CR> 1 <CR> -1 <CR> -1 <CR>."

Step 6. Enter "N <CR>."

Step 9. Enter "0 <CR> 1 <CR>." Enter appropriate response for correction sequence.

Step 10. Enter "1 <CR> 1 <CR>." Enter appropriate response for correction sequence.

Step 11. Enter "1 <CR> .5 <CR>." Enter appropriate response for correction sequence.

Step 12. Enter "5 <CR>."

Step 13. Enter "0 <CR> 0 <CR>." Enter appropriate response for correction sequence.

Step 14. Enter "0 <CR>."

Step 15. Enter "4 <CR>."

Step 16. Enter ".01 <CR>."

Step 17. Enter "10 <CR>." Ensure that the constraint given in step 14 of GTRESP User's Guide is met. That constraint is (TF-TZ)/(DT*FR) = 100

In this case

(4-0)/(.01*10)=40

so the constraint is met.

Step 18. Enter "3 <CR>."

Step 19. Enter "11 <CR> 14 <CR> 2 <CR>." This causes the error e(t), the input r(t), and the state variable $x_2(t)$, respectively, to be plotted as desired from the problem statement.

The program will now run to completion resulting in the output shown in Figure 3.

The output begins with the program name and problem identification. The $\underset{\sim}{A}$, $\underset{\sim}{B}$, and $\underset{\sim}{C}$ matrices are shown for reference followed by the feedback coefficients and the controller gain. Next the initial conditions and time parameters are listed.

The second page of output lists in tabular form the value of time t and the corresponding values of the output y(t), the control u(t), and all of the state variables. The user has no control over the variables output in this form.

The third page of output is the graph itself. Below the graph is enough information to properly interpret the plotted data.

```
GRAPHICAL TIME RESPONSE
PROBLEM IDENTIFICATION -> EXAMPLE
*************************************************

THE A MATRIX

0          1
-1         -1


THE B MATRIX

0          1

THE C MATRIX

1          1

FEEDBACK COEFFICIENTS

1          .5

GAIN = 5


INITIAL CONDITIONS, X(0)

0          0

INITIAL TIME = 0              FINAL TIME = 4

TIME STEP = .01               FREQUENCY OF OUTPUT = 10
*************************************************
```

Figure 3 GTRESP Output

Figure 3 (Cont.)

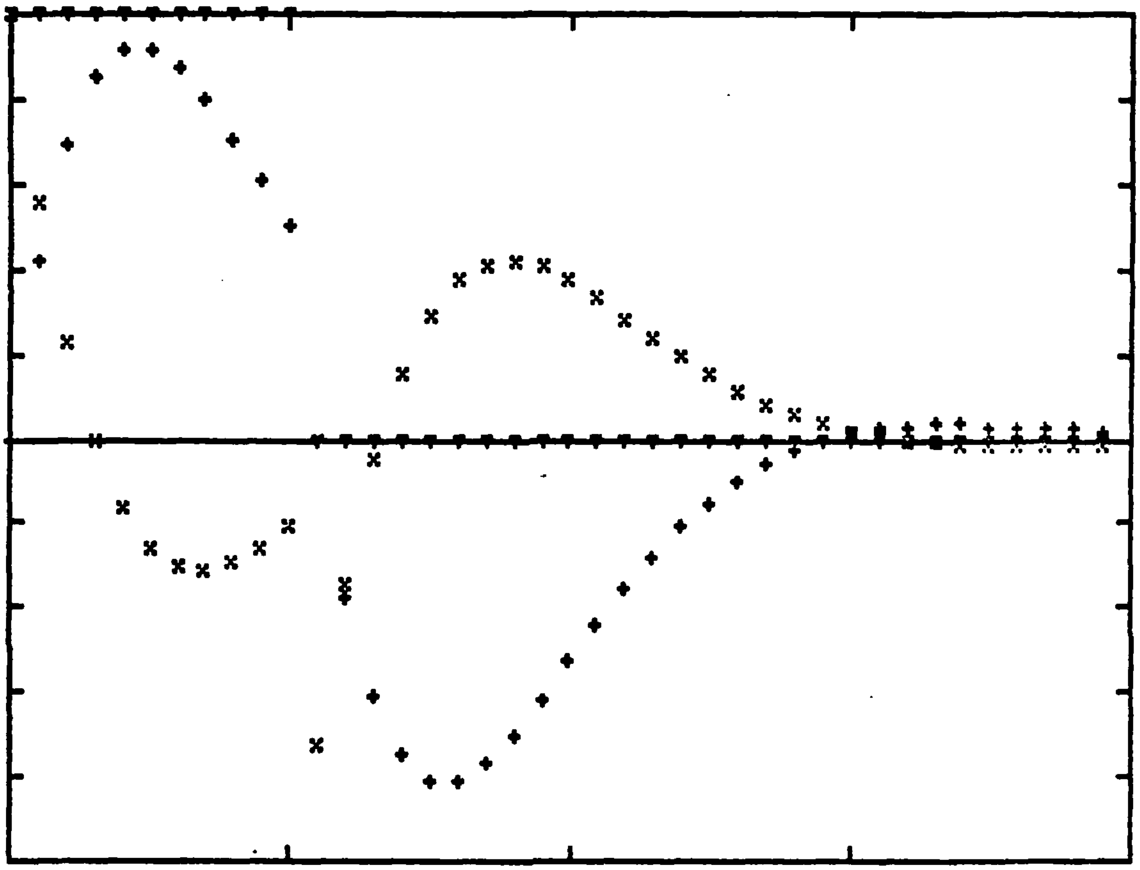
GRAPHICAL TIME RESPONSE
PROBLEM IDENTIFICATION -> EXAMPLE

T	Y(T)	U(T)	X1(T)	X2(T)
0	0	25	0	0
.1	2.19944778	19.2237187	.111064751	2.08838303
.2	3.04846675	14.3947762	.39362276	3.454344
.3	5.0265394	10.4766595	.782796787	4.24374261
.4	5.81339897	7.39775741	1.22749807	4.5859009
.5	6.28493902	5.0653327	1.6889279	4.59601112
.599999998	6.51052439	3.37643244	2.13890263	4.37162176
.699999996	6.55142505	2.22609087	2.55813861	3.99328644
.799999994	6.46012031	1.51321108	2.93459526	3.52552505
.899999991	6.28025128	1.14451516	3.26194266	3.01830862
.999999989	6.04703255	1.03693376	3.53819394	2.50883861
1.09999999	3.62037346	-18.1932806	3.65693877	-.0365653122
1.19999998	1.69860535	-13.1375299	3.5564066	-1.85780124
1.29999998	.258794762	-8.91302849	3.30641664	-3.04762187
1.39999998	-.768858826	-5.48709854	2.96369824	-3.73255707
1.49999998	-1.4543045	-2.79648601	2.57289891	-4.02720341
1.59999998	-1.86407253	-.759576606	2.16790317	-4.03197569
1.69999998	-2.05893263	.714071108	1.77330415	-3.83223678
1.79999997	-2.09253403	1.71656348	1.40590864	-3.49844268
1.89999997	-2.01080576	2.33653976	1.07618985	-3.08699561
1.99999997	-1.85192291	2.65573162	.78963026	-2.64155317
2.09999996	-1.64667353	2.7468857	.547919246	-2.19459277
2.19999996	-1.41908918	2.67275259	.34998815	-1.76907733
2.29999996	-1.18722972	2.48587785	.192878584	-1.38010831
2.39999996	-.964037042	2.22896883	.0724495098	-1.03648655
2.49999996	-.758194579	1.93564872	-.0160649082	-.742129671
2.59999995	-.574947468	1.6314455	-.0776307299	-.497316738
2.69999995	-.416853701	1.33489664	-.117104954	-.299748746
2.79999995	-.284448821	1.05867972	-.139023066	-.145425754
2.89999995	-.176816196	.810704287	-.147465518	-.0293506781
2.99999995	-.0920620427	.595121516	-.145986564	.053924521
3.09999994	-.0276993776	.413224971	-.137590611	.109891233
3.19999994	.0190515208	.264229347	-.124743259	.14379478
3.29999994	.0510366439	.145923933	-.109406217	.160442861
3.39999994	.0710053207	.0552049217	-.0930872894	.16409261
3.49999993	.081500273	-.0115045112	-.0768984685	.158398742
3.59999993	.0847905445	-.05793434	-.0616168085	.146407353
3.69999993	.082838023	-.0877345728	-.0477441939	.130582217
3.79999993	.0772893216	-.104315006	-.0355633192	.112352641
3.89999993	.0694859387	-.110744402	-.0251881781	.0946741168
3.99999992	.0604867868	-.109696615	-.0166081408	.0770949276

Figure 3

Figure 3 (Cont.)

PROBLEM IDENTIFICATION - EXAMPLE
** GRAPHICAL TIME RESPONSE **



ABSCISSA -> TIME AXIS
ORDINATE -> RESPONSE MAGNITUDE
TIC MARKS SHOW INTERVALS OF UNITY
THE PLOT FRAME LIMITS ARE:
ABSCISSA, 0 TO 4
ORDINATE, -5 TO 5

SYSTEM RESPONSE

VARIABLE	SYMBOL
X2	+
ERRORX	
INPUTT	

Figure 3

IV. TRANSFER FUNCTION PROGRAMS

The two transfer function programs discussed in this section are modified versions of programs of the same names found in [Ref. 1].

The Frequency Response program (FRESP), discussed in part A of this section, determines and plots the frequency response of a transfer function over a specified range of frequencies. The output may take the form of rectangular Bode plots or a polar Nyquist plot or both as desired.

The Root Locus program (RTLOC), discussed in part B, calculates and plots the root locus of a transfer function for a specified range of gains. It is also possible to enlarge a small rectangular section of the root locus for more detail.

A. FRESP

1. Frequency Response Program (FRESP)

a. Introduction

The response of a system as a function of frequency is a very important characteristic of that system. Some common ways of graphically displaying the frequency response of a system include the use of amplitude and phase Bode plots and Nyquist diagrams. These are valuable tools for system analysis.

b. Description of Program

The FRESP program [Ref. 1: pp. 105-113] is used to determine the frequency response of a rational transfer function G(s) of the form

$$G(s) = K\frac{A(s)}{B(s)}$$

where

$$A(s) = a_1 + a_2 s + \ldots + a_M s^{M-1} + s^M$$

$$B(s) = b_1 + b_2 s + \ldots + b_N s^{N-1} + s^N$$

The output may take the form of Bode plots or a Nyquist diagram or both in addition to tabular data.

FRESP gives the user the option of supplying discrete frequency values or allowing the program to linearly or logarithmically interpolate frequency values between two limit values. The complex number $G(j\omega)$ can be computed for each frequency value. $G(j\omega)$ can be written in rectangular form as

$$G(j\omega) = R(\omega) + jX(\omega)$$

where $R(\omega)$ and $X(\omega)$ are real values. The magnitude and phase of $G(j\omega)$ can then be written as

$$G(j\omega) = [R^2(\omega) + X^2(\omega)]^{\frac{1}{2}}$$

$$\arg G(j\omega) = \arctan X(\omega)/R(\omega)$$

The FRESP program uses the subroutines PROOT, PVAL, and SEMBL. The subroutines MAXI, PHNOM, GRAPH, and SPLIT that appear in the FORTRAN version do not appear in the BASIC version. They are incorporated into the main program and into various plotting subroutines written for the specific microcomputer system used.

PROOT. This subroutine uses a modified Bairstow method for root extraction to determine the roots of a polynomial with real coefficients. It is used to determine the roots of the numerator and denominator of G(s) when they are entered in polynomial form.

SEMBL. This subroutine determines the coefficients of a polynomial from the roots of the polynomial. It is used to determine the polynomial coefficients of the numerator and denominator of G(s) when they are entered in the factored form. This subroutine together with PROOT provides the feature that G(s) may be entered in either factored or coefficient form and it will appear in the output in both forms.

PVAL. This subroutine is used to evaluate a polynomial A(s) with real coefficients for

s = PR + jPI

PVAL is executed twice for each frequency value used, once for the numerator of the transfer function and again for the denominator.

c. Program Translation Problems

In addition to the programing considerations discussed in section II, an output anomaly was traced to an apparent oversight in the subroutine PVAL. It was discovered that a zero order numerator over a second order denominator with a free s such as

1/s(s+10)

was plotted as

1/(s+10)

The reason for this was that PVAL treated a zero order polynomial and a first order polynomial both as a first order polynomial which in this case resulted in the free s in the denominator being cancelled. The reason for this can be seen by examining the portion of FORTRAN code from PVAL in [Ref. 1: p. 164] repeated below:

```
      P=CMPLX(A(NN+1),0.)
      DO 100 J=1,NN
  100 P=P*S+A(NN+1-J)
      VR=REAL(P)
```

In this portion of code NN represents the order of the polynomial being evaluated and the intention is that the DO loop be executed a number of times equal to the order of the polynomial. But because of the nature of the DO loop, if the polynomial is of zero order (i.e. NN equals zero) the loop will be executed once just as if the polynomial was of first order [Ref. 3]. One way to correct this problem is to modify the section of PVAL as shown here:

```
      P=CMPLX(A(NN+1),0.)
      IF (NN) 110,110,90
   90 DO 100 J=1,NN
  100 P=P*S+A(NN+1-J)
  110 VR=REAL(P)
```

The essence of the modification is to skip the DO loop if the polynomial has order zero. Translating this modification into the BASIC version of FRESP solved the problem.

2. FRESP User's Guide

Push "return" after each input.

STEP 1:

SCREEN PROMPT:

```
FREQUENCY RESPONSE PROGRAM (FRESP)

THIS PROGRAM OBTAINS AND PLOTS THE FREQUENCY
OF A RATIONAL TRANSFER FUNCTION OVER A SPECIFIED
RANGE OF FREQUENCIES.  BOTH RECTANGULAR BODE
PLOTS AS WELL AS A POLAR NYQUIST PLOT CAN BE
OBTAINED.

PROBLEM IDENTIFICATION ->
```

REMARKS:

Label the problem. Type in any appropriate combination of letters, numbers, and/or symbols, excluding commas and colons. This input is limited to 255 characters.

STEP 2:

SCREEN PROMPT:

```
INPUT THE GAIN ->
```

REMARKS:

Enter the transfer function gain.

STEP 3:

SCREEN PROMPT:

```
'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR NUMERATOR ,A(S) ->
```

REMARKS:

Choose the preferred method of entering the numerator polynomial of the transfer function, G(s).

STEP 4A (assumes coefficient form chosen):

SCREEN PROMPT:

```
INPUT POLYNOMIAL COEFFICIENTS

INPUT COEFFICIENTS IN ASCENDING PWRS OF S

INPUT COEFFICIENT OF S^0 ->
```

REMARKS:

Enter the coefficient along with its algebraic sign as prompted. The program assumes that the coefficient of the highest power of s is one.

STEP 4B (assumes factored form chosen):

SCREEN PROMPT:

```
INPUT POLYNOMIAL FACTORS

FOR FACTOR 1
REAL PART ->
```

IMAGINARY PART ->

REMARKS:

Enter the factor with its algebraic sign. Do not enter the associated root. (e.g. For "(s-1)" enter "-1".) After the real part of the factor is entered, the program asks for the imaginary part allowing for input of quadradic factors. If a non-zero imaginary part is entered, the program automatically enters its complex conjugate.

STEP 5:

SCREEN PROMPT:

'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR DENOMINATOR, B(S) ->

REMARKS:

The denominator is entered just as described for the numerator in steps 3 and 4.

STEP 6:

SCREEN PROMPT:

MINIMUM FREQUENCY VALUE ->

REMARKS:

Enter the minimum frequency value in radians per second of the frequency range of interest.

STEP 7:

SCREEN PROMPT:

MAXIMUM FREQUENCY VALUE ->

REMARKS:

Enter the maximum frequency value in radians per second of the frequency range of interest.

STEP 8:

SCREEN PROMPT:

NUMBER OF FREQUENCY VALUES TO BE USED ->

REMARKS:

Enter an integer from 1 to 200. Since the horizontal resolution capability of the graphics on the microcomputer system used is 191 pixels, any number of values greater than 191 adds no more detail to the Bode plots. For the Nyquist diagram the greater the number of values the greater the detail due to the nature of the plot. If a list of discrete frequency values will be supplied (i.e. option 1 in step 9) enter the number of frequencies in that list.

STEP 9:

SCREEN PROMPT:

```
0 = LOGARITHMIC INTERPOLATION
1 = DISCRETE VALUES SUPPLIED
2 = LINEAR INTERPOLATION
```

CHOOSE ONE ->

REMARKS:

Make a choice by entering the associated number. This step allows the user to choose the method used by the program to obtain discrete frequency values used to generate data points. LOGARITHMIC INTERPOLATION produces data points that appear equally spaced on a logarithmic scale between the minimum and maximum frequency values entered in steps 6 and 7. With this choice a Bode and/or Nyquist plot may be obtained. LINEAR INTERPOLATION produces data points that appear equally spaces on a linear scale. Only a Nyquist plot may be obtained with this choice. DISCRETE VALUES SUPPLIED allows the user to supply a list of discrete frequency values of interest. No plots may be obtained, however, with this option. Below is a table summarizing the available graphical output for each choice.

CHOICE	BODE	NYQUIST
0	YES	YES
1	NO	NO
2	NO	YES

STEP 10 (assumes choice 1 in step 9):

SCREEN PROMPT:

TYPE IN THE DISCRETE FREQUENCY VALUES,
(number entered in step 8) VALUES NEEDED
FREQ (1)?

REMARKS:

Enter the list of frequency values as prompted. When the last value has been entered the program will run to completion outputting data similar to Figure 4 with the exception of the plots.

STEP 11 (assumes choice 0 in step 9):

SCREEN PROMPT:

BODE PLOT? (Y/N) ->

REMARKS:

The program outputs tabular data regardless of the choice made here.

STEP 12 (assumes choice 0 or 2 in step 9):

SCREEN PROMPT:

NYQUIST PLOT? (Y/N) ->

REMARKS:

The program outputs tabular data regardless of the choice made here.

After this step, the program runs to completion.

See sample output in Figure 4.

3. FRESP Example

It is desired to know the frequency response of the transfer function

$$G(s) = \frac{8(0.5 + s)}{4 + 6s + 3s^2 + s^3}$$

The frequency range of interest is ω = 0.1 rad/sec to 100 rad/sec. A Bode plot of the magnitude and phase is desired but a Nyquist plot is not. The plots should be generated from 100 values of frequency logarithmically spaced from 0.1 to 100.

Referring to the steps described in the previous section, FRESP User's Guide, this problem would be entered as follows:

STEP 1. Enter "EXAMPLE CR " where <CR> = carriage return.

STEP 2. Enter "8 <CR>."

STEP 3. Enter "F,1 <CR>."

STEP 4B. Enter ".5 <CR> 0 <CR>."

STEP 5. Enter "P,3 <CR> 4 <CR> 6 <CR> 3 <CR>."

STEP 6. Enter ".1 <CR>."

STEP 7. Enter "100 <CR>."

STEP 8. Enter "100 <CR>."

STEP 9. Enter "0 <CR>."

STEP 11. Enter "Y <CR>."

STEP 12. Enter "N <CR>."

The program will then continue to completion producing the output seen in Figure 4.

The output begins with the program name and problem identification. The transfer function gain is given followed by the transfer function numerator and denominator each listed as coefficients in ascending powers of s and then as the real and imaginary parts of the roots.

The second page of output is tabular data. It consists of the radian frequency and the transfer function's corresponding real and imaginary parts, magnitude, and phase in radians and in degrees. Although 100 frequency values were generated and their associated data points plotted, tabular data appears only for every other data point. To reduce unnecessary output and for formatting purposes the following scheme is used for tabular data.

NUMBER OF FREQUENCIES REQUIRED FROM STEP 8	TABULAR DATA PRINTED FOR
0 to 50	every freq.
51 to 100	every other freq.
101 to 150	every 3rd freq.
151 to 200	every 4th freq.

The next two pages of output are the Bode plots for amplitude and phase. The plots are headed with the problem identification and type of plot and below each plot is sufficient information to allow proper interpretation of the plotted data. Note that the phase angles are normalized to always remain between -180 and +180 degrees.

This is the end of the output generated from the example as it was input since a Nyquist plot was not desired. A Nyquist plot is included as the last page of Figure 4 for the sake of completeness.

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - EXAMPLE
*************************************************

GAIN=8

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

   .5
   1

NUMERATOR ROOTS ARE
        REAL PART               IMAGINARY PART

        -.5                     0




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

   4
   6
   3
   1

DENOMINATOR ROOTS ARE
        REAL PART               IMAGINARY PART

        -1                      -1.73205081
        -1                      1.73205081
        -1                      0

*************************************************
```

Figure 4 FRESP Output

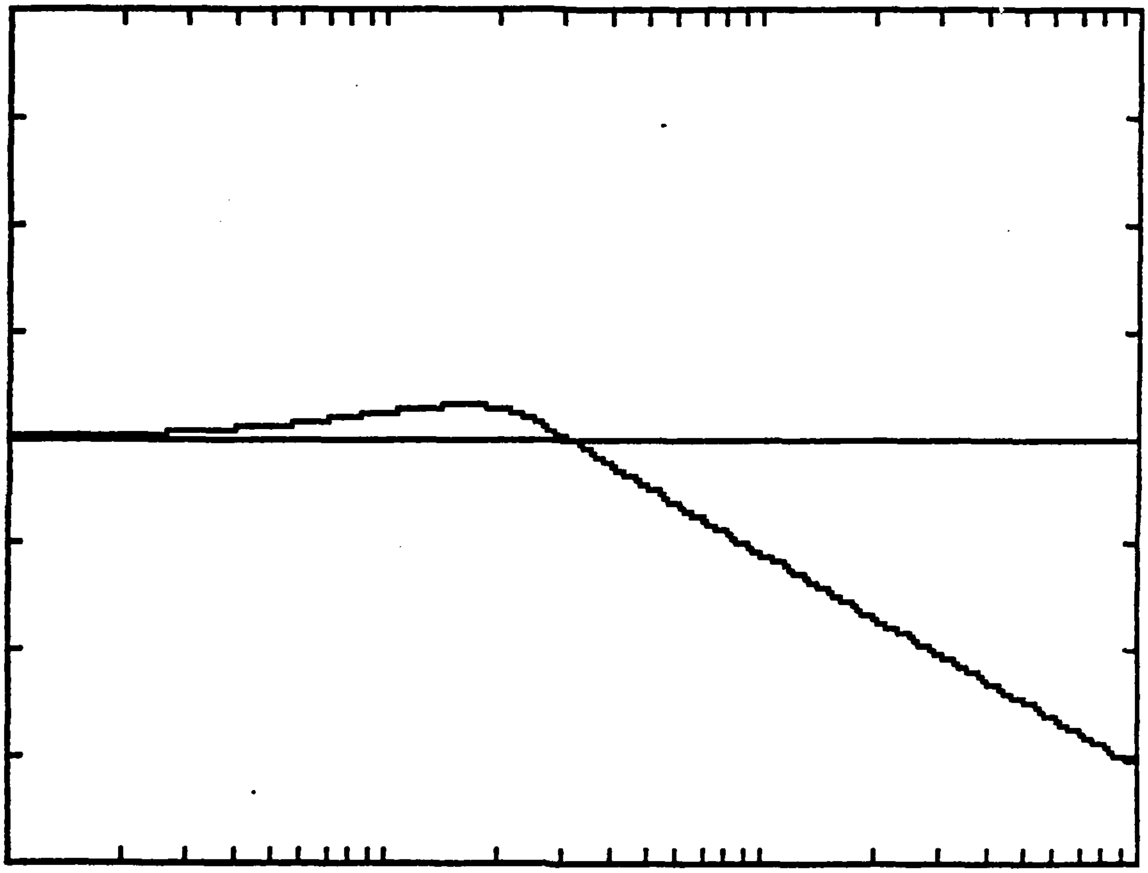
Figure 4 (Cont.) PROBLEM IDENTIFICATION - EXAMPLE

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.107226722	1.01705827	.0516283768	1.01836782	.0507189222	2.90598123
.123284674	1.02247107	.0586342013	1.02415089	.0572828479	3.2820666
.141747416	1.02956848	.0663192682	1.03170223	.0643257561	3.68559566
.162975084	1.03885084	.074600284	1.04152594	.0716873348	4.1073832
.187381743	1.05094983	.0832923195	1.05424531	.0790890087	4.53146802
.215443469	1.06665059	.0920537766	1.07061542	.0860884104	4.93250435
.247707636	1.08690827	.100308178	1.09152705	.0920269474	5.27275758
.284803588	1.11285044	.107136157	1.11799565	.0959760558	5.4990249
.327454917	1.14575167	.111131384	1.1511286	.096691838	5.54003621
.376493582	1.18695955	.110216475	1.19206571	.0925906297	5.3050542
.43287613	1.23774461	.101419568	1.24189277	.0817563658	4.68429638
.497702359	1.29903767	.0806188318	1.30153689	.0619809349	3.55124725
.572236769	1.37100815	.0422676713	1.37165954	.0308198662	1.76584889
.657933228	1.45241093	-.0208852383	1.45256108	-.0143787124	-.823839829
.756463332	1.53955702	-.118060565	1.54407711	-.0765349743	-4.38513258
.869749008	1.62457894	-.260601164	1.6453479	-.159056477	-9.11326809
1.00000001	1.69230769	-.461538466	1.75411604	-.266252052	-15.2551243
1.149757	1.71476725	-.732787826	1.86478008	-.403850578	-23.1389419
1.32194116	1.64366872	-1.07486971	1.96392254	-.579143836	-33.1825094
1.51991109	1.4103691	-1.45016618	2.02289964	-.799309724	-45.7970901
1.74752841	.965794058	-1.7451803	1.99459581	-1.06534155	-61.0395965
2.00923302	.380509559	-1.7966967	1.83654746	-1.36209719	-78.0424484
2.31012972	-.134536079	-1.5559564	1.56176192	-1.65704706	-94.9418367
2.65608781	-.423812809	-1.16877489	1.24324255	-1.91866312	-109.931338
3.05385554	-.50737737	-.803954307	.950670461	-2.13377179	-122.256162
3.51119177	-.478006616	-.52956775	.713394931	-2.30506561	-132.070578
4.0370173	-.406613557	-.342879268	.531884177	-2.44102701	-139.860595
4.64158888	-.32894126	-.220992452	.396282748	-2.55002432	-146.105683
5.33669929	-.259100135	-.142546166	.295723333	-2.63862774	-151.182287
6.13590734	-.201047626	-.0921935098	.221178189	-2.71163856	-155.3655
7.05480239	-.154626892	-.05981062	.165791393	-2.77251189	-158.853287
8.1113084	-.118278908	-.0389127642	.124515475	-2.82375481	-161.789291
9.32603358	-.090161962	-.0253780175	.0936654855	-2.86722005	-164.279667
10.7226724	-.0685723775	-.0165836577	.0705491932	-2.90430709	-166.404598
12.3284676	-.0520722615	-.0108538543	.0531914145	-2.9360967	-168.226009
14.1747418	-.0395004317	-7.11247156E-03	.040135662	-2.96344106	-169.792726
16.2975086	-.029941405	-4.66519619E-03	.0303026696	-2.98702466	-171.143967
18.7381745	-.0226835036	-3.0622238E-03	.0228892671	-3.0074061	-172.311739
21.5443472	-.0171782888	-2.01116278E-03	.0172956174	-3.0250474	-173.322511
24.7707639	-.0130054896	-1.32142716E-03	.0130724492	-3.04033486	-174.198418
28.4803591	-9.84425637E-03	-8.68521676E-04	9.88249531E-03	-3.05359432	-174.958129
32.7454921	-7.45026918E-03	-5.70986204E-04	7.47211725E-03	-3.06510255	-175.617503
37.6493587	-5.63781716E-03	-3.75450085E-04	5.65030488E-03	-3.07509595	-176.190083
43.2876135	-4.26591907E-03	-2.46911164E-04	4.2730587E-03	-3.08377726	-176.687485
49.7702365	-3.22764824E-03	-1.62396323E-04	3.23173108E-03	-3.09132096	-177.119708
57.2236776	-2.44196098E-03	-1.06818684E-04	2.44429615E-03	-3.09787757	-177.495374
65.7933236	-1.84746146E-03	-7.02659939E-05	1.84879722E-03	-3.10357722	-177.821939
75.6463342	-1.39765541E-03	-4.62235779E-05	1.39841956E-03	-3.10853252	-178.105858
86.9749019	-1.05734283E-03	-3.04086657E-05	1.05778001E-03	-3.11284111	-178.352722
100.000002	-7.99879835E-04	-2.00051985E-05	8.00129964E-04	-3.11658766	-178.567383

Figure 4

Figure 4 (Cont.)

PROBLEM IDENTIFICATION - EXAMPLE
** BODE PLOT (AMPLITUDE) **

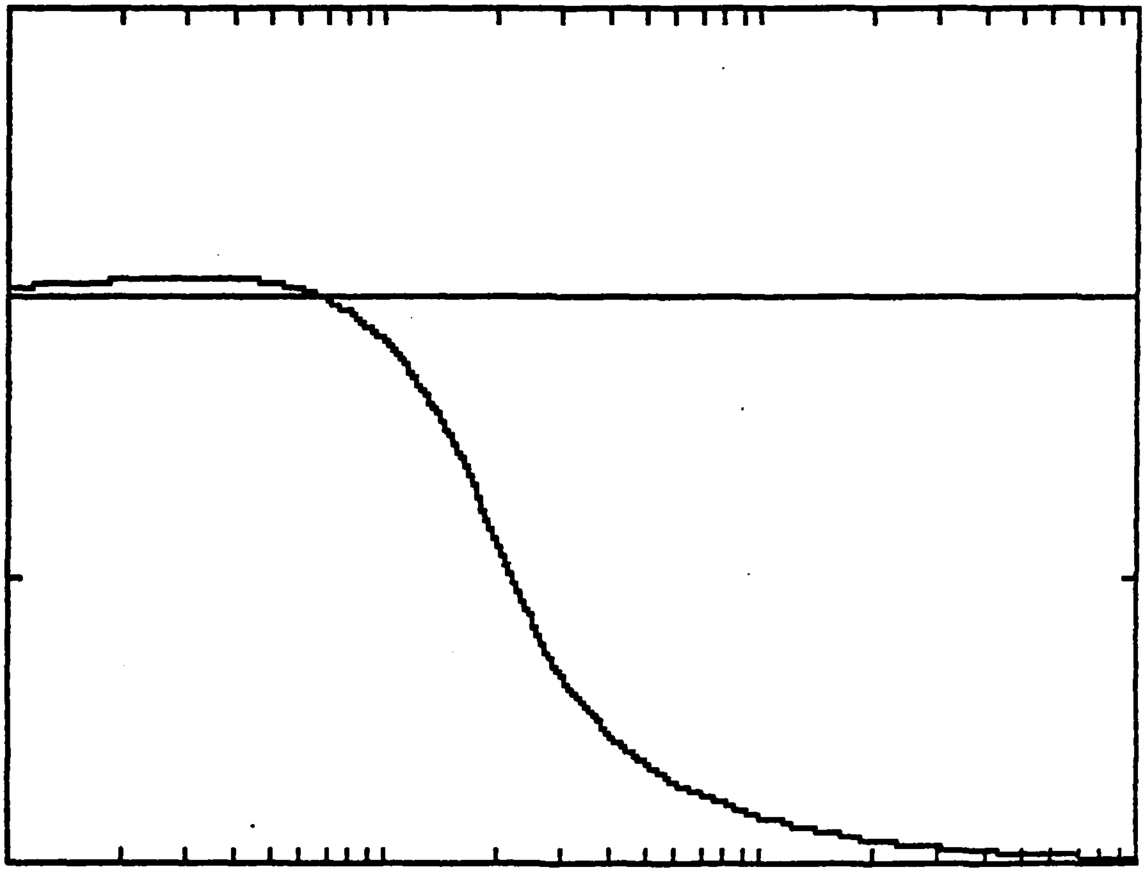


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-80 DECIBELS

Figure 4

Figure 4 (Cont.)

PROBLEM IDENTIFICATION - EXAMPLE
XX BODE PLOT (PHASE) XX



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 90 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Figure 4

Figure 4 (Cont.)

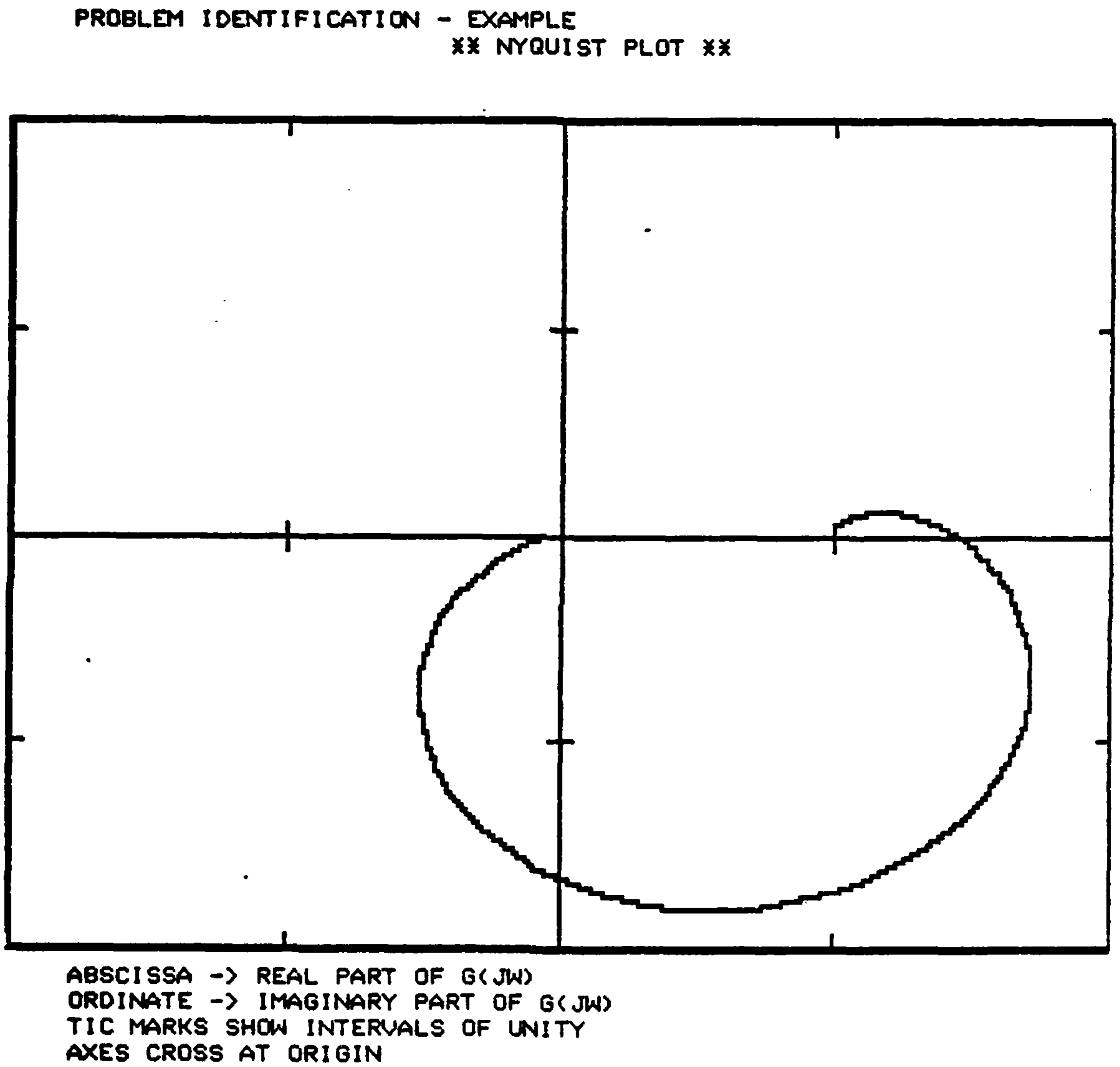


Figure 4

B. RTLOC

1. Root Locus Program (RTLOC)

a. Introduction

When analyzing a system it is helpful to know the location of the closed loop poles in the s-plane. These closed loop poles are the roots of the characteristic equation and determine the basic characteristics of the transient response of a closed loop system. A root locus plot is a plot of the roots of the characteristic equation, usually as a function of the gain of the transfer function, and therefore it is a valuable tool for system analysis.

b. Description of Program

The RTLOC program [Ref. 1: pp. 114-121] calculates and plots the roots of the equation

$$1 + KG(s) = 0$$

as a function of K. G(s) is assumed to be a rational function of the form

$$G(s) = \frac{N(s)}{D(s)}$$

and the root locus becomes the locus of roots of $D_K(s)$ as K varies where

$$D_K(s) = D(s) + KN(s)$$

RTLOC uses an algebraic plus linear progression scheme to vary K to give reasonable spacing of the roots. $D_K(s)$ is obtained for each value of K and the subroutine PROOT is used to calculate its roots.

The scheme used to calculate values of K assumes that K takes on only positive values. If K is to range through negative value, the value of smaller magnitude (less negative) must be used as the minimum value. The routine starts at the maximum (less negative) value and becomes increasingly negative until the lower limit is reached. If both positive and negative values are desired then two separate runs must be made with only positive and only negative values.

The RTLOC program has a feature that allows the user to specify a range of s and ω values around an area of the root locus plot that is of interest. That area is then enlarged and more closely spaced values of K are generated giving more detail to the plot.

The major subroutines used are PROOT and SEMBL. The subroutine SPLIT used in the FORTRAN version is replaced by additional coding in the main program and by various plotting subroutines.

PROOT. This subroutine uses a modified Bairstow method for root extraction to determine the roots of a polynomial with real coefficients. It is used to determine the roots of the numerator and denominator of G(s) when it is entered in polynomial form. It is also used to determine the roots of $D_K(s)$ for each value of K generated.

SEMBL. This subroutine determines the coefficients of a polynomial from the roots of the polynomial. It is used to determine the polynomial coefficients of the numerator and denominator of G(s) when they are entered in the factored form. This subroutine together with PROOT provides the feature that G(s) may be entered in either factored or coefficient form. It appears in the output in both forms.

c. Program Translation Problems

In addition to the programing considerations discussed in section II, two problems were encountered. When a range of negative gains is entered the program calculates only one value of K due to the logic of line 229 in the FORTRAN code. This works correctly only for positive ranges of gain. (Note that K is represented by the variable G in the computer program.) In the BASIC version coding was added to test the sign of the range of gains and modify the logic to correctly handle the negative case.

The second problem was encountered when using gains of very small magnitude necessary in using this program for w'-plane analysis. The schemes used to generate values of K for a reasonable spacing of the roots when plotted worked fine for the magnitudes usually encountered in s-plane analysis but was not flexible enough to handle gains of much smaller magnitudes (e.g. 0 to 1E-3). To add this necessary flexibility the following FORTRAN lines used to generate values of gain G,

```
227 G=1.15*(G+SIGNG*0.05)
228 G=1.04*(G+SIGNG*0.02)
```

were modified by replacing the constant values 0.05 and 0.02 with the variables D1 and D2, respectively, where

```
D1=ABS(GMIN-GMAX)/700
D2=ABS(GMIN-GMAX)/1500
```

so that the gain increment was a function of the range of gains of interest.

2. <u>RTLOC User's Guide</u>

Push "return" after each input.

STEP 1:

SCREEN PROMPT:

```
ROOT LOCUS PROGRAM (RTLOC)

THIS PROGRAM PLOTS THE ROOT LOCUS OF A
DESCRIBED SYSTEM.

PROBLEM IDENTIFICATION ->
```

REMARKS:

Label the problem. Type in any appropriate combination of letters, numbers, and/or symbols, excluding commas and colons. This input is limited to 255 characters.

STEP 2:

SCREEN PROMPT:

```
INPUT THE RANGE OF GAINS (MIN,MAX) ->
```

REMARKS:

Enter the range of transfer function gains of interest. Note that the minimum gain is the gain of lowest absolute value.

STEP 3:

SCREEN PROMPT:

```
'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR NUMERATOR, N(S) ->
```

REMARKS:

Choose the preferred method of entering the numerator polynomial of the transfer function G(s).

STEP 4A (assumes coefficient form chosen):

SCREEN PROMPT:

```
INPUT POLYNOMIAL COEFFICIENTS

INPUT COEFFICIENTS IN ASCENDING PWRS OF s OR
w'

INPUT COEFFICIENT OF S^0 ->
```

REMARKS:

Enter the coefficient along with its algebraic sign as prompted. The program assumes that the coefficient of the highest power of s is one.

STEP 4B (assumes factored form chosen):

SCREEN PROMPT:

```
INPUT POLYNOMIAL FACTORS

FOR FACTOR 1
REAL PART ->

IMAGINARY PART ->
```

REMARKS:

Enter the factor with its algebraic sign. Do not enter the associated root. (e.g. For "(s-1)" enter "-1".) After the real part of the factor is entered the program asks for the imaginary part allowing for input of quadradic factors. If a non-zero imaginary part is entered the program automatically enters its complex conjugate.

STEP 5:

SCREEN PROMPT:

```
'KEY' = P FOR POLYNOMIAL COEFFICIENT FORM
'KEY' = F FOR POLYNOMIAL FACTORED FORM

INPUT 'KEY', ORDER FOR DENOMINATOR, D(S) ->
```

REMARKS:

The denominator is entered just as described for the numerator in steps 3 and 4.

STEP 6:

SCREEN PROMPT:

```
WOULD YOU LIKE TO LOOK AT ONLY A PART OF THE
ROOT LOCUS?  (Y/N) ->
```

REMARKS:

Enter an "N" for the option of viewing the root locus for the entire range of gains entered in STEP 2. Enter a "Y" for the option of viewing only a portion of the root locus defined by minimum and maximum values of sigma and ω of interest to the user. Usually, it is best to view the entire root locus first then decide if any section needs to be enlarged to reveal more detail and rerun RTLOC to view only that portion.

STEP 7 (assumes "Y" entered in step 6):

SCREEN PROMPT:

ENTER SIGMA MIN, SIGMA MAX ->

REMARKS:

Enter the minimum and maximum values of sigma (i.e. the real axis) of the portion of the root locus plot of interest.

STEP 8 (assumes "Y" entered in step 6):

SCREEN PROMPT:

ENTER OMEGA MIN, OMEGA MAX ->

REMARKS:

Enter the minimum and maximum values of omega (i.e. the imaginary axis) of the portion of the root locus plot of interest.

STEP 9:

SCREEN PROMPT:

PRINT OUT OF GAIN DATA? (Y/N) ->

REMARKS:

This gives the user the option to suppress the print out of the gain data if it is not needed. This saves time and paper if only the root locus plot is desired. After this step the program runs to completion.

3. RTLOC Example

It is desired to know the root locus for the open loop transfer function G(s) given by

$$G(s) = \frac{1.2 + s}{s(8 + 9s + s^2)}$$

for gain K of from 0 to 30. Referring to the steps described in section 2, RTLOC User's Guide, this problem would be entered as follows:

STEP 1. Enter "EXAMPLE <CR>" where <CR> = carriage return.

STEP 2. Enter "0,30 <CR>."

STEP 3. Enter "F,1 <CR>."

STEP 4B. Enter "1.2 <CR> 0 <CR>."

STEP 5. Enter "F,3 <CR> 0 <CR> 0 <CR> 1 <CR> 0 <CR> 8 <CR> 0 <CR>."

STEP 6. Enter "N <CR>."

STEP 9. Enter "Y <CR>."

The program will then run to completion producing the output seen in Figure 5.

The output begins with the program name and problem identification. Next the transfer function numerator and denominator are listed as coefficients in ascending powers of s and then as the real and imaginary parts of the roots (i.e. open loop zeros for the numerator and open loop poles for the denominator). The minimum and maximum values of gain are listed next.

If the option to view only a part of the root locus is selected, this fact is stated next followed by the ranges of sigma and omega that are chosen. (See Figure 6)

The next several pages of output contain the gain data which may be suppressed with the appropriate response in step 9. The data point number is listed with the value of gain and the real and imaginary parts of the corresponding roots of the open loop system.

The last page of output contains the root locus plot itself. It begins with the problem identification and heading and ends with sufficient data listed to allow proper interpretation of the plot.

Figure 6 shows the output from the same example problem with the gain data suppressed and the option to view only a part of the root locus chosen.

```
ROOT LOCUS
PROBLEM IDENTIFICATION - EXAMPLE
**************************************************

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  1.2
  1


OPEN-LOOP ZEROS
       REAL PART               IMAGINARY PART

       -1.2                    0




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  0
  8
  9
  1


OPEN-LOOP POLES
       REAL PART               IMAGINARY PART

       -1                      0
       -8                      0
       0                       0

       MIN GAIN                MAX GAIN
       0                       30

**************************************************
```

Figure 5 RTLOC Output

Figure 5 (Cont.)

1 GAIN = 0

ROOTS ARE

REAL PART	IMAG. PART
-1	0
-8	0
0	0

2 GAIN = .0492857143

ROOTS ARE

REAL PART	IMAG. PART
-7.40891563E-03	0
-.998580111	0
-7.99401098	0

3 GAIN = .105964286

ROOTS ARE

REAL PART	IMAG. PART
-.0159695125	0
-.996917641	0
-7.98711284	0

4 GAIN = .171144643

ROOTS ARE

REAL PART	IMAG. PART
-.0258689696	0
-.994965313	0
-7.97916572	0

5 GAIN = .246102054

ROOTS ARE

REAL PART	IMAG. PART
-.037328049	0
-.992664433	0
-7.97000752	0

6 GAIN = .332303076

ROOTS ARE

REAL PART	IMAG. PART
-.0506084588	0
-.989941249	0
-7.95945029	0

7 GAIN = .431434251

ROOTS ARE

REAL PART	IMAG. PART
-.0660224557	0
-.986701776	0
-7.94727577	0

8 GAIN = .545435103

ROOTS ARE

REAL PART	IMAG. PART
-.0839456946	0
-.982824229	0
-7.93323008	0

9 GAIN = .676536083

ROOTS ARE

REAL PART	IMAG. PART
-.104834979	0
-.978147608	0
-7.91701742	0

10 GAIN = .82730221

ROOTS ARE

REAL PART	IMAG. PART
-.972453845	0
-7.89829238	0
-.129253772	0

Figure 5

Figure 5 (Cont.)

```
11      GAIN = 1.00068326                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -.965438583            0
                                     -7.87665069            0
                                     -.15791073             0
12      GAIN = 1.20007146                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -.956660582            0
                                     -7.8516177             0
                                     -.191721725            0
13      GAIN = 1.42936789                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -.945447526            0
                                     -7.82263429            0
                                     -.231918189            0
14      GAIN = 1.69305879                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.78903912            0
                                     -.280258299            0
                                     -.930702581            0
15      GAIN = 1.99630332                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.75004614            0
                                     -.339507185            0
                                     -.910446682            0
16      GAIN = 2.34503453                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.70471555            0
                                     -.414824269            0
                                     -.880460187            0
17      GAIN = 2.74607543                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.65191587            0
                                     -.520147205            0
                                     -.827936925            0
18      GAIN = 3.20727245                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.59027312            0
                                     -.704863441            -.101133243
                                     -.704863441            .101133243
19      GAIN = 3.73764904                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.51810122            0
                                     -.740949392            -.218123799
                                     -.740949392            .218123799
20      GAIN = 4.34758211                        ROOTS ARE
                                     REAL PART              IMAG. PART
                                     -7.43330403            0
                                     -.783347989            -.29701938 4
                                     -.783347989            .29701938 4
```

Figure 5

Figure 5 (Cont.)

21	GAIN = 5.04900514	ROOTS ARE	
		REAL PART	IMAG. PART
		-7.33323262	0
		-.833383692	-.362882638
		-.833383692	.362882638
22	GAIN = 5.85564162	ROOTS ARE	
		REAL PART	IMAG. PART
		-7.2144689	0
		-.892765554	-.420657402
		-.892765554	.420657402
23	GAIN = 6.78327358	ROOTS ARE	
		REAL PART	IMAG. PART
		-7.07248055	0
		-.963759725	-.471271626
		-.963759725	.471271626
24	GAIN = 7.85005033	ROOTS ARE	
		REAL PART	IMAG. PART
		-6.9010354	0
		-1.0494823	-.513427903
		-1.0494823	.513427903
25	GAIN = 9.07684359	ROOTS ARE	
		REAL PART	IMAG. PART
		-6.69112225	0
		-1.15443888	-.543259781
		-1.15443888	.543259781
26	GAIN = 10.4876559	ROOTS ARE	
		REAL PART	IMAG. PART
		-6.42872813	0
		-1.28563594	-.552076334
		-1.28563594	.552076334
27	GAIN = 12.1100899	ROOTS ARE	
		REAL PART	IMAG. PART
		-6.08944969	0
		-1.45527515	-.518280319
		-1.45527515	.518280319
28	GAIN = 13.9758891	ROOTS ARE	
		REAL PART	IMAG. PART
		-5.62140192	0
		-1.68929904	-.360139054
		-1.68929904	.360139054
29	GAIN = 16.1215582	ROOTS ARE	
		REAL PART	IMAG. PART
		-2.6433642	0
		-4.84656548	0
		-1.51007033	0
30	GAIN = 18.5890777	ROOTS ARE	
		REAL PART	IMAG. PART
		-3.80149761	-1.23137034
		-3.80149761	1.23137034
		-1.39700479	0

Figure 5

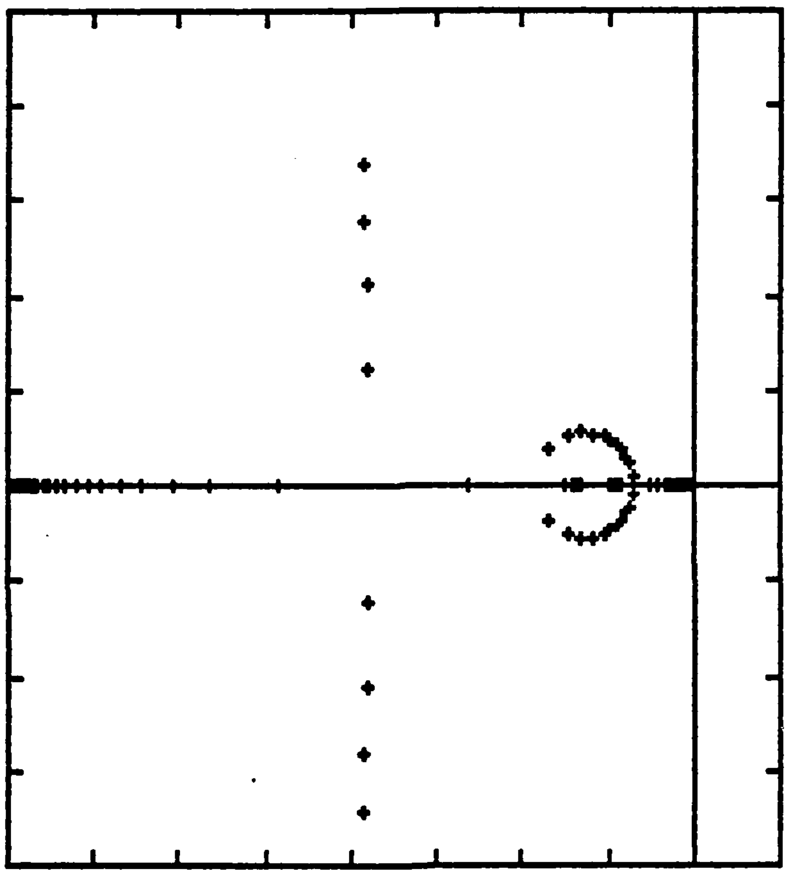
Figure 5 (Cont.)

31	GAIN = 21.426725	ROOTS ARE	
		REAL PART	IMAG. PART
		-3.82838189	-2.11787032
		-3.82838189	2.11787032
		-1.34323622	0
32	GAIN = 24.6900195	ROOTS ARE	
		REAL PART	IMAG. PART
		-3.8449682	-2.79855914
		-3.8449682	2.79855914
		-1.3100636	0
33	GAIN = 28.4428081	ROOTS ARE	
		REAL PART	IMAG. PART
		-3.85636161	-3.41216555
		-3.85636161	3.41216555
		-1.28727678	0

Figure 5

Figure 5 (Cont.)

PROBLEM IDENTIFICATION - EXAMPLE
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -8 TO 1
ORDINATE, -4 TO 5

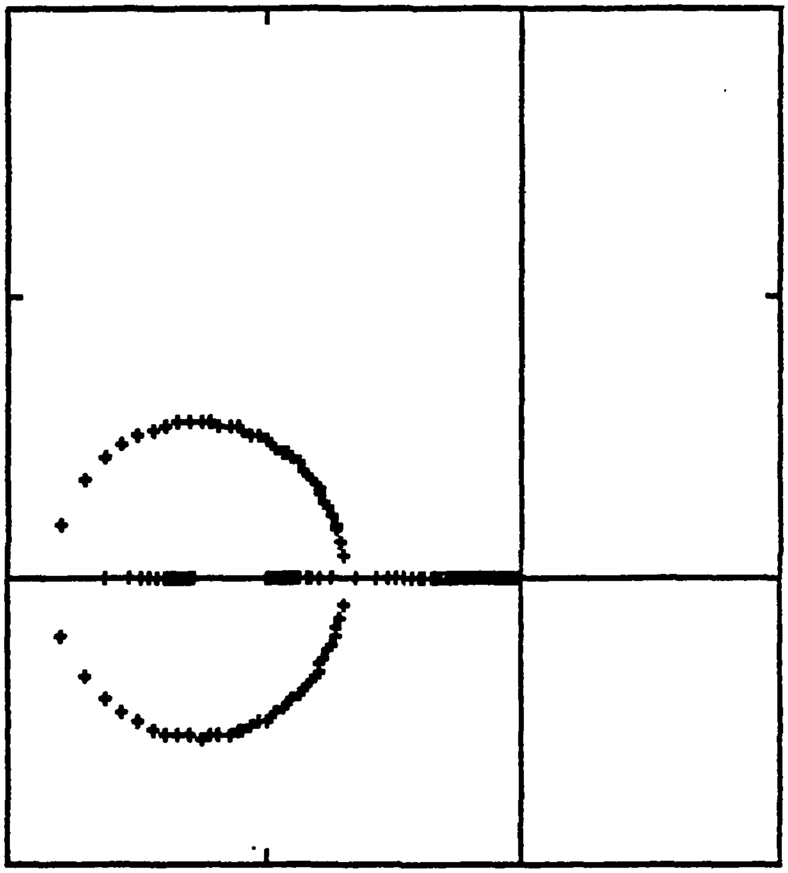
Figure 5

```
ROOT LOCUS
PROBLEM IDENTIFICATION - EXAMPLE WITH OPTION
**************************************************

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  1.2
  1

OPEN-LOOP ZEROS
       REAL PART              IMAGINARY PART

       -1.2                   0

DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  0
  8
  9
  1

OPEN-LOOP POLES
       REAL PART              IMAGINARY PART

       -1                     0
       -8                     0
       0                      0

       MIN GAIN               MAX GAIN
       0                      30

**************************************************

OPTION TAKEN

SIGMA MIN = -2                SIGMA MAX = 0
OMEGA MIN = -1                OMEGA MAX = 1
```

Figure 6 RTLOC Output with Option

Figure 6 (Cont.)

PROBLEM IDENTIFICATION - EXAMPLE WITH OPTION
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -2 TO 1
ORDINATE, -1 TO 2

Figure 6

V. w'-PLANE ANALYSIS

Digital control systems are becoming more and more common. Digital control laws have unique characteristics that can only be approximated by using classical techniques in the continuous s domain.

In the w' domain all analog control system design technology transfers completely for digital control system design. An important advantage of the w' domain is that non-minimum phase effects of the sampling and data-hold operations and of sampling rate can be directly accounted for without approximation while using conventional frequency domain design and analysis tools such as root locus and Bode plots. [Ref. 4]

A. BACKGROUND

This s domain is used for continuous system analysis. When a digital system is considered the z, w, or w'domain must be used. These domains are related as shown below.

$$z = e^{sT}$$

$$w = \frac{z-1}{z+1} = \tanh \frac{sT}{2}$$

$$w' = (2/T)\ w$$

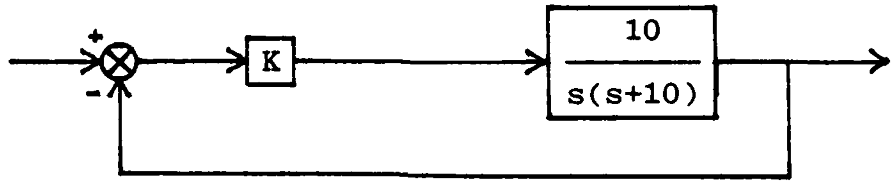
where T is the sampling period and

$$\tanh x = \frac{e^{x} - e^{x}}{e^{x} + e^{x}}$$

In an s-plane root locus plot the region of stability is the left half plane, that is all roots with negative real parts. This region of stability is mapped into a unit circle with its center at the origin in the z-plane. By use of the bilateral transformation shown above, the stability region of the z-plane is mapped back into the left half plane to form the w-plane. The w'-plane takes it a step farther by multiplying the w-plane by the factor 2/T where T is the sampling period. This gives the w'-plane the property that not only is the region of stability the left half plane as in the continuous s-plane but w' approaches s as the sampling period T approaches zero [Ref. 5].

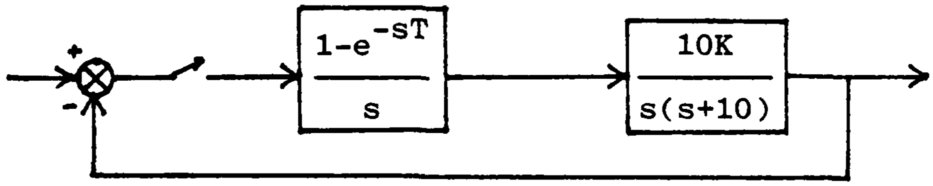
B. TRANSFER FUNCTION APPROACH USING THE w'-PLANE

In this section the second order system



is analyzed using the two transfer function programs discussed in part IV of this thesis. It is then converted to the w'-plane and analyzed for periods of .001 seconds, .01 seconds, and .1 seconds using the same two transfer function programs. These results are used to gain insight into w'-plane analysis.

The system is converted to the w'-plane by adding a sampler and a digital to analog converter in the form of a zero order hold. This modified system is shown below.



The new open loop transfer function is transformed to the z-plane and then to the w'-plane as follows:

$$G(s) = 10K(1-e^{-sT})\ \frac{1}{s^2(s+10)}$$

$$G(z) = 10K(1-z^{-1}) \quad \frac{1}{10} \left[\frac{Tz}{(z-1)^2} - \frac{(1-e^{-10T})z}{10(z-1)(z-e^{-10T})} \right]$$

$$= K \left[\frac{T}{z-1} - \frac{1-e^{-10T}}{10(z-e^{-10T})} \right]$$

For the period

$T=.001$

G(z) reduces to

$$G(z) = \frac{4.98337 \times 10^{-6} \, K(z+.996681)}{(z-1)(z-.990049834)} \tag{1}$$

Similarly, for

$T=.01$

$$G(z) = \frac{4.837418 \times 10^{-4} \, K(z+.9672185)}{(z-1)(z-.904837418)}$$

And for

$T=.1$

$$G(z) = \frac{.03678794412 \, K(z+.718281827)}{(z-1)(z-.3678794412)}$$

Now G(z) is converted to the w' domain using the relationship

$$z = \frac{1 + T/2\ w'}{1 - T/2\ w'}$$

For period

$$T=.001$$

$$z = \frac{1 + .0005\ w'}{1 - .0005\ w'}$$

Substituting into equation (1) gives

$$G(w') = \frac{4.98337 \times 10^{-6}\ K \left(\dfrac{1+.0005w'}{1-.0005w'} + .996681 \right)}{\left(\dfrac{1+.0005w'}{1-.0005w'} - 1 \right) \left(\dfrac{1+.0005w'}{1-.0005w'} - .990049834 \right)}$$

which reduces to

$$G(w') = \frac{-4.156878 \times 10^{-9}\ K\ (w' + 1202819)\ (w' - 2000)}{w'\ (w' + 9.9999)} \qquad (2)$$

Note that the gain is 1.0K which is the same as the s domain gain. Similarly, for

$$T=.01$$

$$z= \frac{1+.005w'}{1-.005w'}$$

$$G(w') = \frac{-4.1625027 \times 10^{-6} K(w'+12002.00418)(w'-200)}{w'(w' + 9.991674985)} \quad (3)$$

And for

$T=.1$

$$z = \frac{1+.05w'}{1-.05w'}$$

$$G(w') = \frac{-.0037882843 K(w' + 121.9858708)(w'-20)}{w'(w' + 9.242343139)} \quad (4)$$

Note again that the gain is 1.0K also in equations (3) and (4).

1. <u>Frequency Response</u>

a. s-Plane

The open loop transfer function of the system in the s domain is

$$G(s) = \frac{10K}{s(s+10)}$$

The frequency response of G(s) with a gain K of 5 is described by the output of the FRESP program shown in Figure 7. From the Bode plots it is seen that the gain margin is infinite and the phase margin is 65.1 degrees.

b. w'-Plane with a Period of .001 Seconds

The open loop transfer function for the equivalent sampled system is represented by equation (2) for a period T of .001 seconds. This transfer function is entered into the FRESP program as if w' were an s. The resulting output in Figure 8 is interpreted as described earlier in the thesis keeping in mind that all references to s are actually referring to w'. From this output it is seen that the gain margin is no longer infinite as it was in the continuous case but it is still quite high at 50 dB. The phase margin has dropped slightly from 65.1 degrees to 64.6 degrees.

c. w'-Plane with a Period of .01 Seconds

If the sampling rate of the system is decreased so that the sampling period is .01 seconds, then the open loop transfer function is represented by equation (3). Inputting the transfer function into FRESP results in the output shown in Figure 9. From the Bode plots the gain margin is found to be 32 dB. This is down from 50 dB for the case of a period of .001 seconds indicating a decrease in stability. The phase margin is 64.5 degrees which is also lower than the previous case but only by one tenth of a degree.

d. w'-Plane with a Period of .1 Seconds

If the period is again increased to .1 seconds, the open loop transfer function is represented by equation

(4). Entering this transfer function into FRESP produces the output seen in Figure 10. From this output the gain margin is found to have decreased to 13.2 dB and the phase margin to 53.7 degrees.

e. Summary of Frequency Response Results

The table below is a brief summary of the gain and phase margins found in each case.

CASE	GAIN MARGIN	PHASE MARGIN
s-plane	infinite	65.1 deg.
w', T=.001	50 dB	64.6 deg.
w', T=.01	32 dB	64.5 deg.
w', T=.1	13.2 dB	53.7 deg.

It can be seen from this table that the continuous case is the most stable. The sampled cases become less stable as the sampling period increases. It is also noticed that the gain margin is more sensitive to changes in the sampling period than the phase margin.

2. Root Locus

Now a comparison is made using the root locus program using the same cases used above.

a. s-Plane

If the open loop transfer function

$$G(s) = \frac{10K}{s(s+10)}$$

is entered into the RTLOC program, the result is the output in Figure 11. The root locus plot shows that the system never becomes unstable at any gain.

b. w'-Plane with a Period of .001 Seconds

If the equivalent sampled transfer function with a sampling period of .001 seconds represented by equation (2) is entered into RTLOC, the output will be that found in Figure 12. From the root locus plot it can be seen that there is a slight tendency for the plot to curve toward the right half plane as it moves further from the real axis. The tendency is so slight that it will take a very large gain to drive the system unstable.

c. w'-Plane with a Period of .01 Seconds

If the sampling period is increased to .01 seconds the transfer function is represented by equation (3). Entering this transfer function into RTLOC results in the output seen in Figure 13. It can be seen that the tendency for the curve to bend toward the unstable right half plane is increased indicating that a lower value of gain than in the previous case will drive the system to instability.

d. w'-Plane with a Period of .1 Seconds

When the sampling period is further increased to .1 seconds represented by equation (4), the resulting RTLOC output is that seen in Figure 14. In the root locus plot for this case the tendency to become unstable is much more pronounced. Here the plot bends into the right half plane within the limited boundaries of the portion plotted. It becomes unstable for values of gain K greater then 24.6.

e. Summary of Root Locus Comparison

It can be seen that the continuous system

$$G(s) = \frac{10K}{s(s+10)}$$

is always stable. When this system is sampled it can be seen by equations (2), (3), and (4) that a zero is added in the right half plane making it a non-minimum phase system. It can also be seen that by decreasing or increasing the sampling period the distance of this zero from the origin increases or decreases respectively. The closer this zero is to the imaginary axis, the greater effect it has on bending the root locus into the right half plane. This effect can be seen by examining the root locus plots in Figures 11 through 14.

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - EX1 S-PLANE
*************************************************

GAIN=50

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  1





DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  0
  10
  1

DENOMINATOR ROOTS ARE
       REAL PART             IMAGINARY PART

       0                     0
       -10                   0

*************************************************
```

Figure 7 s-Plane Frequency Response Example

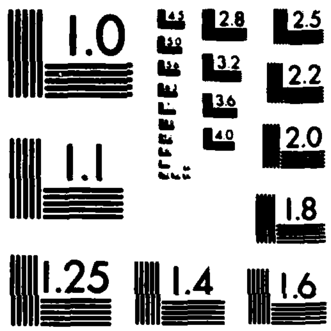
Figure 7 (Cont.) PROBLEM IDENTIFICATION - EX1 S-PLANE

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.107226722	-.499942519	-46.6248066	46.6274869	-1.58151864	-90.6143755
.123284674	-.499924016	-40.5503782	40.5534598	-1.58312422	-90.7063685
.141747416	-.499899558	-35.2669256	35.2704684	-1.58497017	-90.8121336
.162975084	-.49986723	-30.6713897	30.6754627	-1.58709244	-90.9337311
.187381743	-.499824502	-26.6741303	26.6788128	-1.58953236	-91.073528
.215443469	-.499768027	-23.1971769	23.2025599	-1.59233739	-91.2342445
.247707636	-.499693392	-20.1727084	20.1788964	-1.59556207	-91.4190054
.284803588	-.499594763	-17.54173	17.5488429	-1.59926904	-91.6313988
.327454917	-.49946444	-15.2529223	15.2610977	-1.60353017	-91.8755439
.376493582	-.499292266	-13.2616408	13.2710365	-1.60842796	-92.1561667
.43287613	-.499064843	-11.5290451	11.5398417	-1.61405698	-92.4786859
.497702359	-.498764522	-10.0213413	10.0337455	-1.62052558	-92.8493093
.572236769	-.498368069	-8.70912349	8.72337108	-1.62795771	-93.2751396
.657933228	-.497844948	-7.5668005	7.58316026	-1.63649501	-93.7642907
.756463332	-.497155095	-6.57209774	6.5908749	1.64629891	-94.326013
.869749008	-.49624608	-5.70562399	5.72716381	-1.65755295	-94.9708225
1.00000001	-.495049503	-4.95049503	4.97518594	-1.67046503	-95.7106301
1.149757	-.49347653	-4.29200717	4.32028294	-1.68526942	-96.5588596
1.32194116	-.491412428	-3.71735478	3.74969502	-1.70222842	-97.5305392
1.51991109	-.488710161	-3.2153865	3.25231425	-1.72163305	-98.6423427
1.74752841	-.485183205	-2.77639666	2.81847143	-1.74380222	-99.9125431
2.00923302	-.480598168	-2.39194839	2.43975239	-1.76907957	-101.360829
2.31012972	-.474668375	-2.05472607	2.10884075	-1.79782663	-103.007915
2.65608781	-.467050509	-1.75841517	1.81938454	-1.83041085	-104.874854
3.05385554	-.457347608	-1.49760721	1.56588448	-1.86718669	-106.981955
3.51119177	-.445123138	-1.26772665	1.34360168	-1.90846789	-109.347194
4.0370173	-.429931824	-1.06497394	1.14848199	-1.95448982	-111.984058
4.64158888	-.411372483	-.886275139	.977093108	-2.00536225	-114.898834
5.33669929	-.38916454	-.729223288	.826568597	-2.06101572	-118.087544
6.13590734	-.363241793	-.591993609	.694550959	-2.12114917	-121.532939
7.05480239	-.333844701	-.473216233	.579125106	-2.18519089	-125.20226
8.1113084	-.301580297	-.371802282	.478735431	-2.25228766	-129.046623
9.32603358	-.267415569	-.286740946	.392086033	-2.32133517	-133.002755
10.7226724	-.232584424	-.21690901	.318033069	-2.39105392	-136.997347
12.3284676	-.198419696	-.160944331	.2554867	-2.46010143	-140.953479
14.1747418	-.166155293	-.117219273	.203341927	-2.5271982	-144.797843
16.2975086	-.136758201	-.0839135631	.160450278	-2.59123991	-148.467164
18.7381745	-.110835455	-.0591495479	.125631075	-2.65137337	-151.912558
21.5443472	-.0886275129	-.0411372468	.0977093094	-2.70702684	-155.101268
24.7707639	-.0700681732	-.028286642	.0755624445	-2.75789927	-158.016045
28.4803591	-.0548768595	-.0192683173	.0581613081	-2.8039212	-160.652908
32.7454921	-.0426523898	-.0130254234	.0445969507	-2.84520239	-163.018147
37.6493587	-.0329494891	-8.75167342E-03	.0340919437	-2.88197823	-165.125248
43.2876135	-.0253316228	-5.85193332E-03	.0259987738	-2.91456245	-166.992187
49.7702365	-.0194018304	-3.89827973E-03	.0197895833	-2.94330952	-168.639273
57.2236776	-.0148167935	-2.58927671E-03	.0150413339	-2.96858686	-170.087559
65.7933236	-.0112898382	-1.71595499E-03	.0114194986	-2.99075603	-171.357759
75.6463342	-8.58757143E-03	-1.13523630E-03	8.66228157E-03	-3.01016066	-172.469563
86.9749019	-6.52346907E-03	-7.50040406E-04	6.56644572E-03	-3.02711966	-173.441243
100.000002	-4.95049495E-03	-4.95049488E-04	4.97518586E-03	-3.04192405	-174.289472

Figure 7

AD-A132 945 THE USE OF A MICROCOMPUTER SYSTEM AS AN AID TO CLASSICAL AND DIGITAL CONTROL SYSTEM DESIGN AND ANALYSIS(U) NAVAL POSTGRADUATE SCHOOL MONTEREY CA 2/2

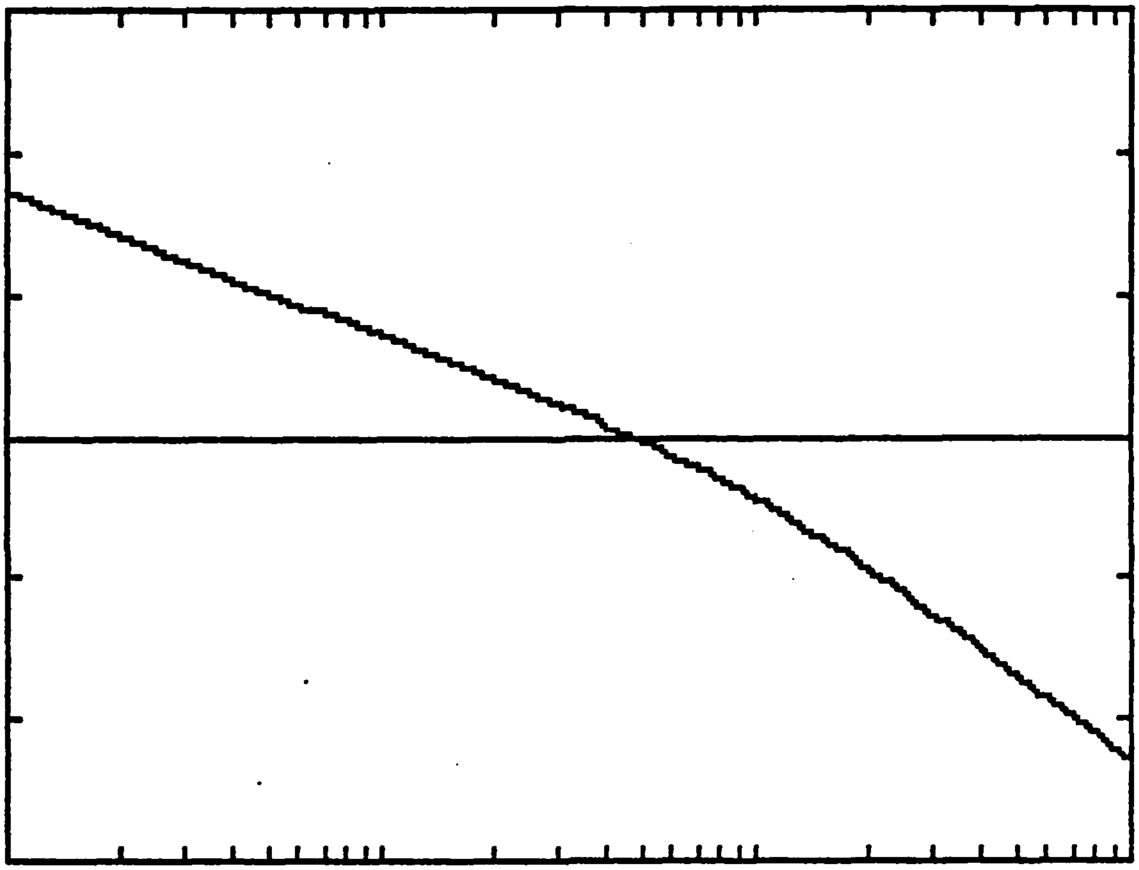
UNCLASSIFIED J D HUMPHREY JUN 83 F/G 9/2 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Figure 7 (Cont.)

PROBLEM IDENTIFICATION - EX1 S-PLANE
** BODE PLOT (AMPLITUDE) **

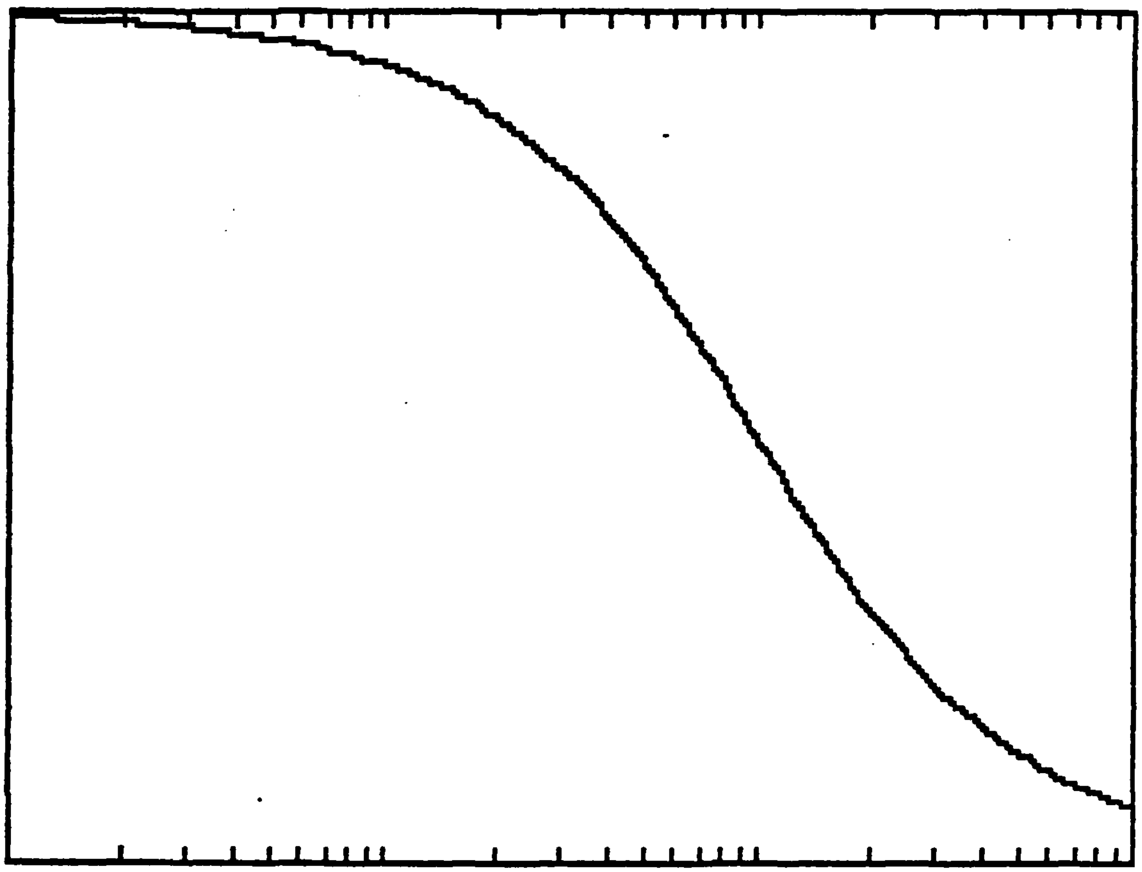


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-60 DECIBELS

Figure 7

Figure 7 (Cont.)

PROBLEM IDENTIFICATION - EX1 S-PLANE
** BODE PLOT (PHASE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = -90 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Figure 7

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001 EXTENDED
**************************************************

GAIN=-2.078439E-08

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  -2.405638E+09
  1200819
  1

NUMERATOR ROOTS ARE
       REAL PART               IMAGINARY PART

       -1202819                0
       2000                    0

DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  0
  9.9999
  1

DENOMINATOR ROOTS ARE
       REAL PART               IMAGINARY PART

       0                       0
       -9.9999                 0

**************************************************
```

Figure 8 w'-Plane Frequency Response Example, T=.001

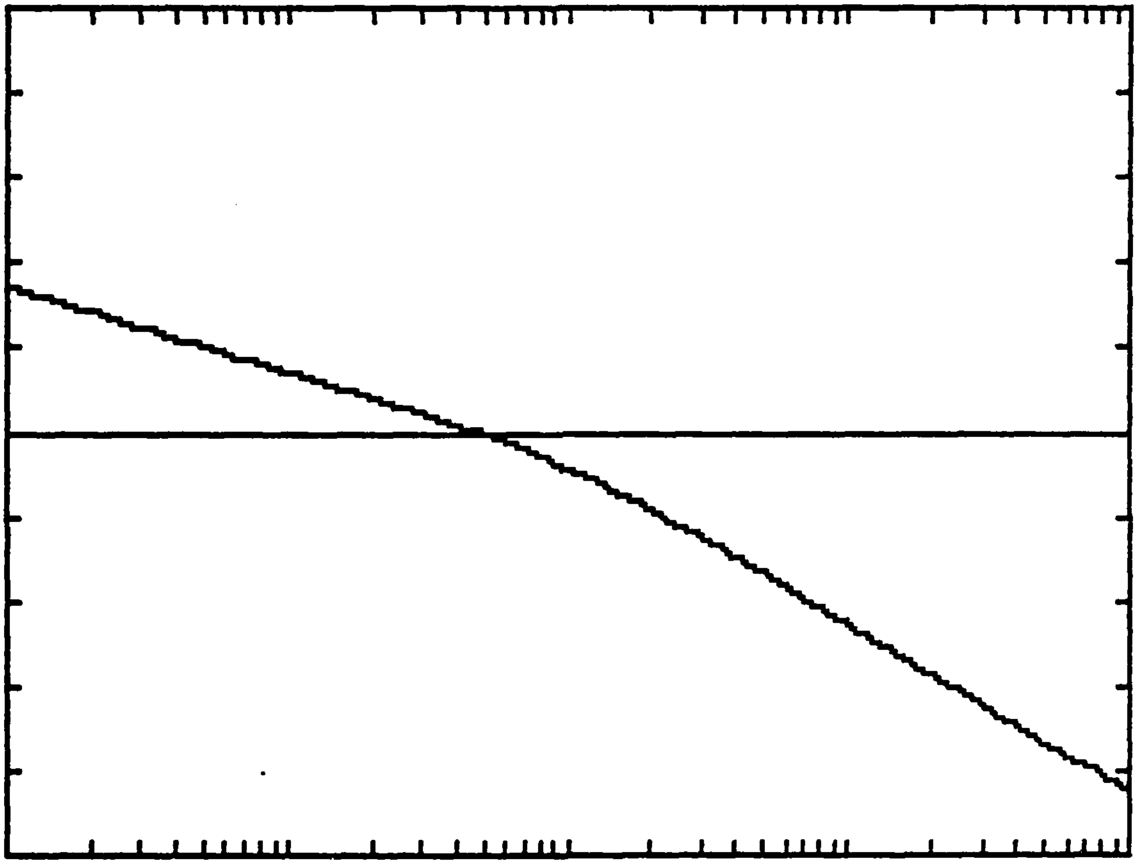
Figure 8 (Cont.) PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001 EXTENDE

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.109749877	-.502442517	-45.5528224	45.5555933	-1.58182581	-90.6319756
.132194115	-.502415237	-37.8166898	37.8200272	-1.58408114	-90.7611959
.159228279	-.502375664	-31.393595	31.3976144	-1.5867975	-90.9163321
.191791026	-.502318262	-26.0605212	26.0653618	-1.59006905	-91.1042785
.23101297	-.502235005	-21.6322985	21.6381279	-1.59400911	-91.3300271
.278255941	-.502114261	-17.955175	17.9621944	-1.59875396	-91.6018875
.335160266	-.501939186	-14.9014781	14.9099294	-1.6044675	-91.9292483
.403701727	-.501685399	-12.3651825	12.3753556	-1.61134656	-92.32339
.48626016	-.501317654	-10.2582291	10.2704714	-1.61962733	-92.7973436
.585702084	-.500785075	-8.50746911	8.52219552	-1.62959269	-93.3683171
.705480234	-.500014404	-7.05212636	7.06983032	-1.64158056	-94.0556715
.849753439	-.498900501	-5.84169077	5.86295598	-1.65599308	-94.8814465
1.02353103	-.497293211	-4.83417109	4.85968216	-1.67330621	-95.8734131
1.23284675	-.494979637	-3.99464784	4.02519766	-1.69407869	-97.0635939
1.48496827	-.491661052	-3.29407918	3.33056876	-1.71895861	-98.4891084
1.78364954	-.486924704	-2.70832265	2.75174622	-1.74868424	-100.192263
2.1544347	-.480213076	-2.21734474	2.26874906	-1.78407386	-102.219939
2.59502423	-.470796148	-1.80459822	1.86500017	-1.82599609	-104.621987
3.12571587	-.457776887	-1.45655107	1.52679426	-1.87530956	-107.447362
3.76493583	-.440116426	-1.16234702	1.24288096	-1.93275848	-110.738943
4.53487854	-.4167883	-.913559865	1.00414347	-1.99831444	-114.523672
5.46227726	-.387025866	-.703966769	.803341916	-2.07346959	-118.801899
6.5793323	-.350693265	-.529223808	.634872905	-2.15601713	-123.530726
7.92482905	-.308654899	-.386325031	.494484456	-2.24489756	-128.623202
9.54548465	-.262928031	-.272830121	.378902923	-2.33771426	-133.941208
11.4975701	-.216412607	-.186052021	.285394063	-2.43148859	-139.314084
13.8488638	-.172211076	-.122546894	.211363185	-2.5231181	-144.56407
16.6810055	-.132845421	-.0781417301	.154123443	-2.60988337	-149.535356
20.0923302	-.0997604401	-.0484083292	.110885129	-2.68981801	-154.115275
24.2012829	-.0732817618	-.0292485321	.0789030623	-2.76184146	-158.241916
29.1505309	-.0529078594	-.0172935105	.0556624389	-2.8256786	-161.899516
35.1119177	-.0377008078	-.0100263967	.0390112746	-2.88166281	-165.107176
42.2924292	-.0266059887	-5.70075876E-03	.0272098748	-2.93051822	-167.906386
50.9413808	-.0186452458	-3.17015952E-03	.018912829	-2.97317316	-170.350621
61.3590735	-.0130013548	-1.71211774E-03	.0131136027	-3.01065846	-172.498065
73.9072212	-9.03396706E-03	-8.84629075E-04	9.0771763E-03	-3.04398135	-174.407347
89.0215097	-6.26175023E-03	-4.23027939E-04	6.27602332E-03	-3.07413774	-176.135181
107.226724	-4.33276499E-03	-1.71309219E-04	4.3361503E-03	-3.1020752	-177.73583
129.154968	-2.99443621E-03	-3.86030236E-05	2.99468503E-03	-3.12870183	-179.261475
155.567616	-2.06778405E-03	2.75163739E-05	2.06796712E-03	3.12828631	179.237667
187.381745	-1.42707452E-03	5.70369751E-05	1.42821389E-03	3.1016462	177.7113
225.701975	-9.84502339E-04	6.69616004E-05	9.86776931E-04	3.07368161	176.109047
271.858828	-6.78999083E-04	6.68301543E-05	6.8228002E-04	3.04348418	174.378861
327.454921	-4.68210766E-04	6.19206743E-05	4.72287509E-04	3.01010615	172.46644
394.420612	-3.22819434E-04	5.50937949E-05	3.27486967E-04	2.97255685	170.315023
475.081023	-2.22557794E-04	4.78502267E-05	2.27643616E-04	2.92981525	167.866108
572.236775	-1.53428024E-04	4.09341491E-05	1.5879472E-04	2.88086856	165.061669
689.261221	-1.05760475E-04	3.46757013E-05	1.11307568E-04	2.82478945	161.848571
830.217582	-7.29133323E-05	2.91841137E-05	7.85370394E-05	2.7608643	158.185929
1000.00002	-5.02650559E-05	2.44556446E-05	5.59986082E-05	2.68877604	154.055574

Figure 8

Figure 8 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001 EXTENDED
** BODE PLOT (AMPLITUDE) **

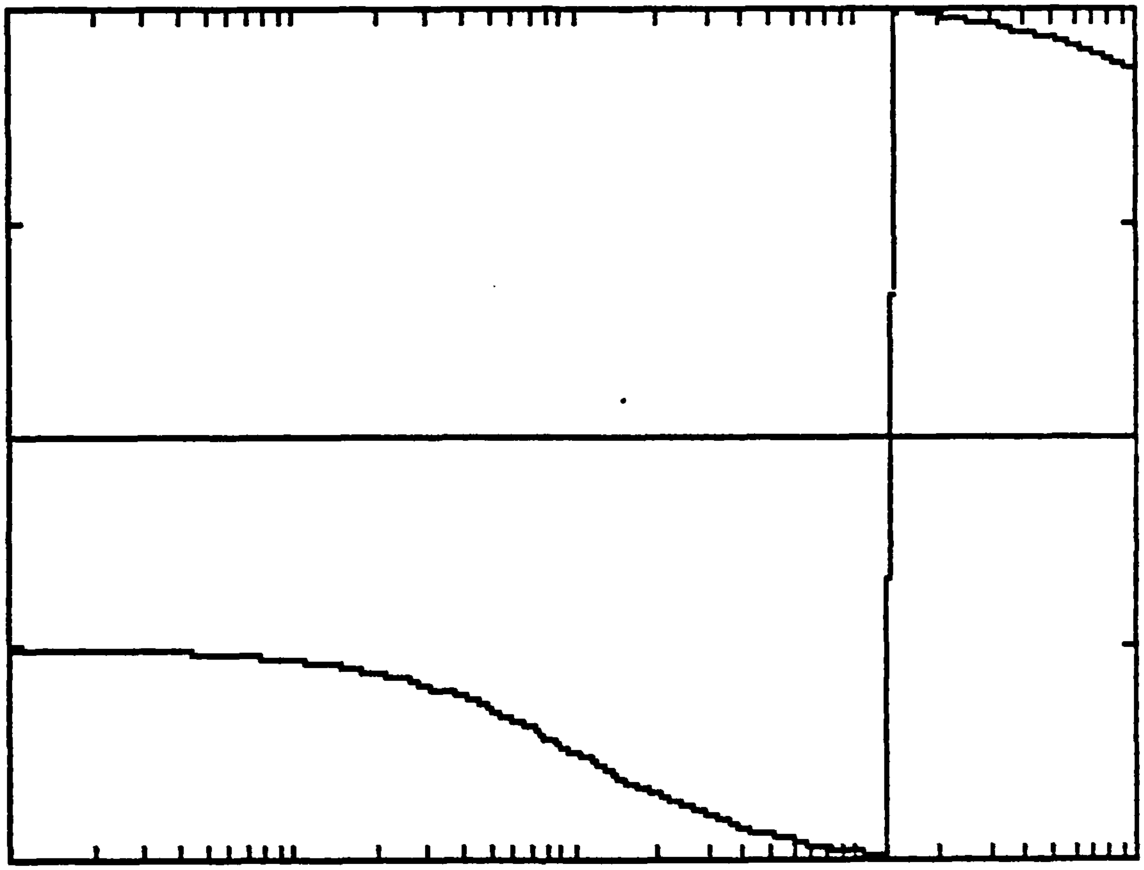


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 1000 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-100 DECIBELS

Figure 8

Figure 8 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001 EXTENDED
** BODE PLOT (PHASE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 1000 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 180 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Figure 8

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01
*************************************************

GAIN=-2.08125135E-05

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  -2400400.84
  11802.0042
  1

NUMERATOR ROOTS ARE
       REAL PART            IMAGINARY PART

       -12002.0042          0
       200                  0



DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  0
  9.99167498
  1

DENOMINATOR ROOTS ARE
       REAL PART            IMAGINARY PART

       0                    0
       -9.99167498          0

*************************************************
```

Figure 9 w'-Plane Frequency Response Example, T=.01

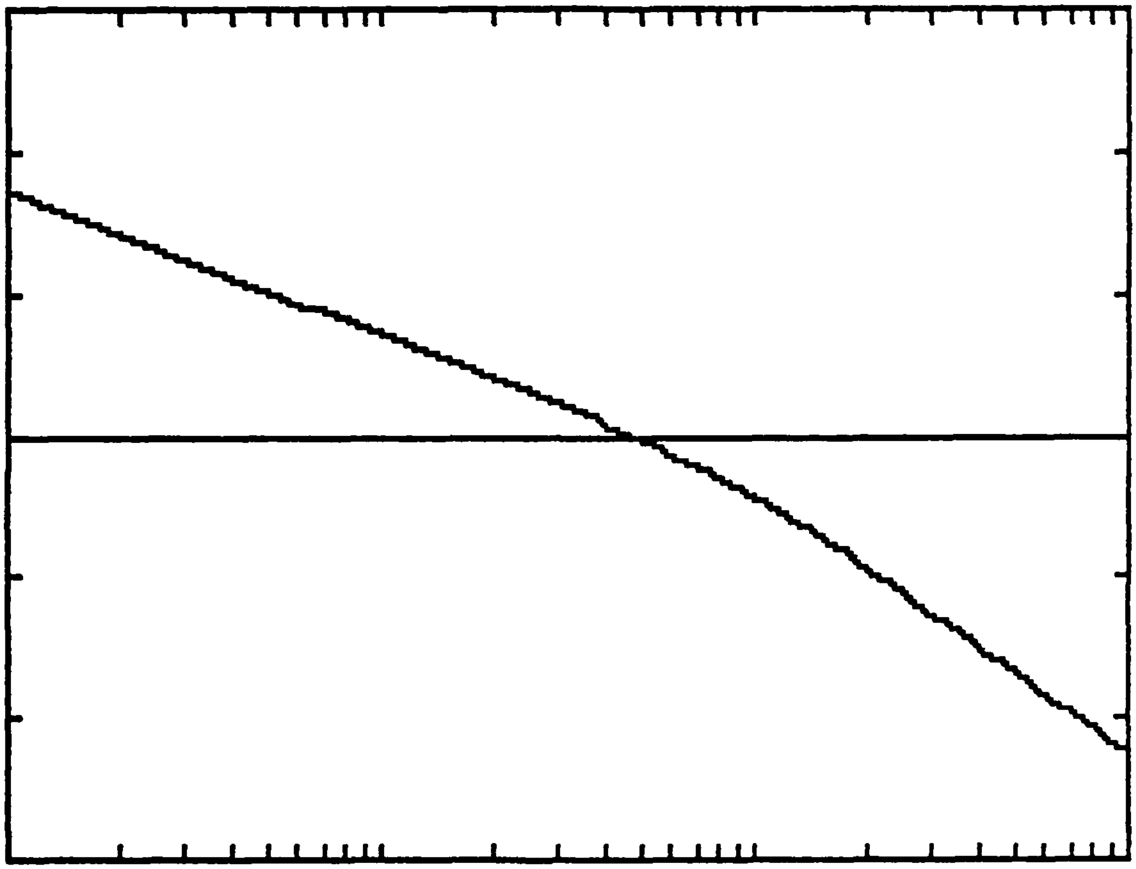
Figure 9 (Cont.) PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.107226722	-.524939547	-46.6245341	46.6274892	-1.58205477	-90.6458936
.123284674	-.524920088	-40.5500649	40.5534623	-1.58374064	-90.7416869
.141747416	-.524894366	-35.2665654	35.2704713	-1.5856789	-90.8527412
.162975084	-.524860366	-30.6709756	30.6754662	-1.58790731	-90.9804198
.187381743	-.524815428	-26.6736542	26.6788168	-1.59046926	-91.1272086
.215443469	-.524756033	-23.1966296	23.2025644	-1.5934146	-91.2959641
.247707636	-.524677539	-20.1720793	20.1789016	-1.5968006	-91.4899677
.284803588	-.52457381	-17.5410067	17.5488488	-1.60069303	-91.7129879
.327454917	-.524436749	-15.2520909	15.2611045	-1.60516741	-91.9693511
.376493582	-.524255672	-13.2606853	13.2710444	-1.61031038	-92.2640214
.43287613	-.524016491	-11.527947	11.5398508	-1.61622129	-92.6026918
.497702359	-.523700643	-10.0200796	10.0337559	-1.62301398	-92.9918844
.572236769	-.523283694	-8.70767398	8.72338307	-1.63081873	-93.439064
.657933228	-.52273353	-7.56513572	7.58317406	-1.63978443	-93.9527606
.756463332	-.522008018	-6.57018639	6.59089081	-1.65008085	-94.5427023
.869749008	-.52105202	-5.70343055	5.72718215	-1.66190113	-95.2199546
1.00000001	-.519793606	-4.9479794	4.97520709	-1.67546416	-95.9970594
1.149757	-.51813936	-4.2091243	4.32030737	-1.69101689	-96.8881655
1.32194116	-.515968635	-3.71405452	3.74972327	-1.70883614	-97.9091336
1.51991109	-.513126826	-3.21161357	3.25234698	-1.72922959	-99.077593
1.74752841	-.509417817	-2.77209106	2.81850943	-1.75253532	-100.412913
2.00923302	-.504596244	-2.38704633	2.43979662	-1.77911889	-101.93604
2.31012972	-.49836076	-2.04916171	2.1088924	-1.809367	-103.66913
2.65608781	-.490350539	-1.75212349	1.81944508	-1.84367592	-105.634087
3.05385554	-.480148485	-1.49052834	1.56595572	-1.88243306	-107.855508
3.51119177	-.467296083	-1.25981191	1.34368586	-1.92598991	-110.351133
4.0370173	-.451325898	-1.05619372	1.14858184	-1.97462499	-113.137719
4.64158888	-.431817202	-.876628211	.97721191	-2.02849743	-116.224383
5.33669929	-.408476439	-.718746596	.826710149	-2.08759426	-119.610383
6.13590734	-.381235751	-.580770561	.694719472	-2.15167917	-123.28218
7.05480239	-.350349774	-.461380989	.579324936	-2.22025484	-127.211277
8.1113084	-.316458228	-.359537358	.47897069	-2.29255366	-131.353696
9.32603358	-.280578464	-.274266592	.392360087	-2.36756996	-135.651815
10.7226724	-.244007236	-.20446504	.318348055	-2.444138	-140.038842
12.3284676	-.208143963	-.148767675	.255843176	-2.5210461	-144.445353
14.1747418	-.174282984	-.105522173	.203738772	-2.59716482	-148.806636
16.2975086	-.143437538	-.0728670818	.160884863	-2.67156109	-153.06923
18.7381745	-.116242624	-.0488750465	.126099635	-2.74357614	-157.19539
21.5443472	-.0929485537	-.0317058744	.0982074138	-2.81285769	-161.164932
24.7707639	-.0734841707	-.0197248894	.0760854428	-2.87935069	-164.974701
28.4803591	-.0575538451	-.0115669167	.0587046732	-2.94325927	-168.636395
32.7454921	-.0447358616	-6.14914618E-03	.0451564981	-3.00499418	-172.173546
37.6493587	-.0345626152	-2.64835976E-03	.034663932	-3.06511716	-175.61834
43.2876135	-.0265759352	-4.59917609E-04	.0265799146	-3.12428864	-179.008617
49.7702365	-.0203593365	8.48006795E-04	.0203769894	3.09996478	177.614962
57.2236776	-.0155527033	1.57682656E-03	.0156324331	3.04055195	174.210856
65.7933236	-.0118553889	1.9329344E-03	.0120119308	2.97997212	170.739887
75.6463342	-9.02265597E-03	2.05532664E-03	9.2537932E-03	2.9176184	167.167281
86.9749019	-6.85893251E-03	2.0361868E-03	7.15478944E-03	2.85301322	163.465675
100.000002	-5.21006089E-03	1.93572135E-03	5.55803487E-03	2.78586306	159.618253

Figure 9

Figure 9 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01
** BODE PLOT (AMPLITUDE) **

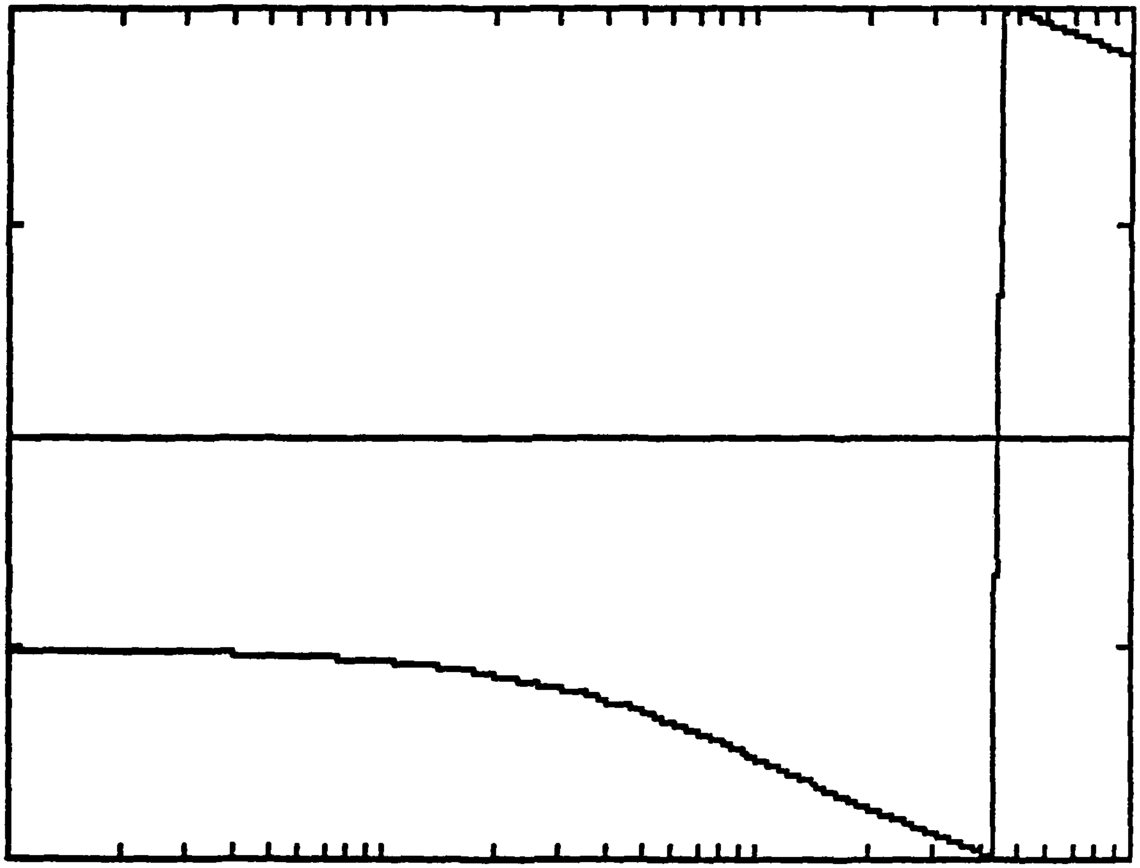


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-60 DECIBELS

Figure 9

Figure 9 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01
** BODE PLOT (PHASE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 180 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Fig re 9

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.1
*************************************************

GAIN=-.0189414215

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  -2439.71742
  101.985871
  1

NUMERATOR ROOTS ARE
       REAL PART             IMAGINARY PART

       -121.985871           0
       20                    0




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  0
  9.24234314
  1

DENOMINATOR ROOTS ARE
       REAL PART             IMAGINARY PART

       0                     0
       -9.24234314           0

*************************************************
```

Figure 10 w'-Plane Frequency Response Example, T=.1

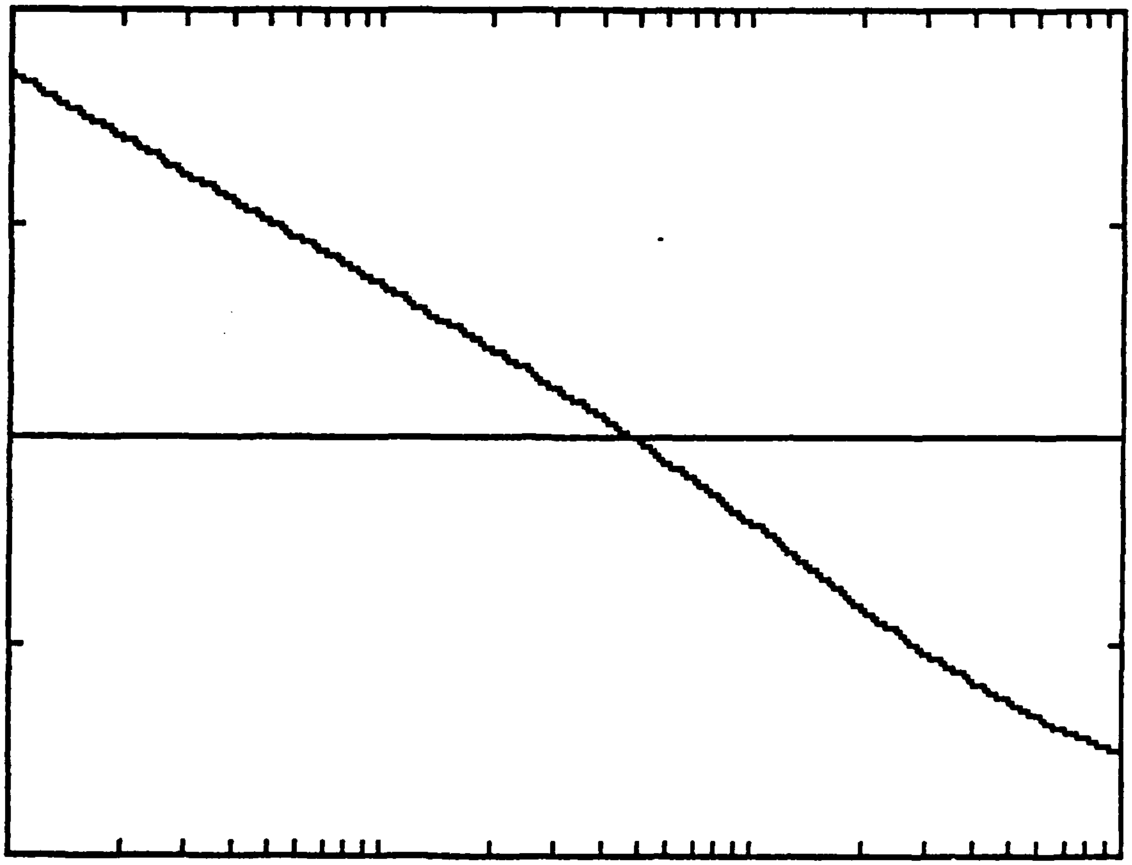
Figure 10 (Cont.) PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.1

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.107226722	-.749901617	-46.6216872	46.6277179	-1.58687981	-90.9215483
.123284674	-.749869949	-40.5467918	40.5537253	-1.58928821	-91.0595392
.141747416	-.749820088	-35.2628023	35.2707736	-1.59205717	-91.218189
.162975084	-.749772759	-30.6666494	30.6758137	-1.59524063	-91.4005381
.187381743	-.749699629	-26.6686807	26.6792163	-1.59890059	-91.6102882
.215443469	-.749602979	-23.1909122	23.2030238	-1.60310826	-91.8513702
.247707636	-.749475252	-20.165507	20.1794298	-1.60794548	-92.1285223
.284803588	-.749306472	-17.5334525	17.5494563	-1.61350621	-92.4471289
.327454917	-.749083476	-15.2434088	15.2618032	-1.61989834	-92.8133714
.376493582	-.748788896	-13.2507081	13.271848	-1.62724569	-93.2343435
.43287613	-.748399844	-11.5164835	11.5407753	-1.63569023	-93.7181801
.497702359	-.747886175	-10.0069112	10.0348196	-1.64539465	-94.274203
.572236769	-.747208246	-8.6925516	8.72460736	-1.65654519	-94.9130817
.657933228	-.746313996	-7.54777607	7.58458359	-1.66935467	-95.6470111
.756463332	-.745135214	-6.55026855	6.59251428	-1.6840659	-96.4899028
.869749008	-.743582787	-5.68059274	5.72905309	-1.70095514	-97.4575856
1.00000001	-.741540723	-4.92181645	4.97736475	-1.72033568	-98.5680092
1.149757	-.738858839	-4.25918654	4.32279798	-1.74256117	-99.8414361
1.32194116	-.735343947	-3.6798489	3.75260159	-1.7680284	-101.300602
1.51991109	-.730749803	-3.17260863	3.25567824	-1.79717901	-102.970809
1.74752841	-.724766284	-2.72772751	2.82237198	-1.83049917	-104.879914
2.00923302	-.717009119	-2.33675582	2.44428514	-1.86851612	-107.058126
2.31012972	-.707012534	-1.99239681	2.11412199	-1.91179005	-109.53754
2.65608781	-.694228885	-1.68840209	1.82555618	-1.96089918	-112.351287
3.05385554	-.678041196	-1.41949479	1.57311962	-2.01641584	-115.532159
3.51119177	-.657796528	-1.1813151	1.35211007	-2.07887135	-119.110597
4.0370173	-.632868475	-.970377373	1.15851403	-2.14870819	-123.111955
4.64158888	-.602754245	-.784021224	.988939817	-2.22622029	-127.553073
5.33669929	-.567202992	-.62033237	.840554272	-2.31148516	-132.438391
6.13590734	-.526356326	-.478007285	.71101473	-2.40429639	-137.756085
7.05480239	-.480862856	-.356146355	.598388931	-2.50410884	-143.474919
8.1113084	-.431916356	-.253986373	.501059694	-2.61000982	-149.5426
9.32603358	-.38117579	-.170619167	.417619304	-2.72072574	-155.886158
10.7226724	-.330561899	-.10476951	.346767674	-2.83466462	-162.414377
12.3284676	-.281976491	-.0546999148	.287233046	-2.94998502	-169.021752
14.1747418	-.237027502	-.0182671504	.237730363	-3.0646771	-175.593126
16.2975086	-.196840011	6.90255476E-03	.196960999	3.10654024	177.991708
18.7381745	-.161992858	.0231913619	.163644509	2.99939605	171.852796
21.5443472	-.132569482	.0327930441	.13656519	2.89909561	166.106002
24.7707639	-.10827854	.0375850315	.114616216	2.80749096	160.85744
28.4803591	-.0885946078	.0390773022	.0968299546	2.72618143	156.198746
32.7454921	-.0728831383	.0384219724	.0823905323	2.65644429	152.2031
37.6493587	-.060493015	.0364590583	.070630502	2.59919126	148.922743
43.2876135	-.0508148715	.0337766111	.0610164784	2.55495334	146.388095
49.7702365	-.0433112758	.0307705931	.0531290506	2.5238862	144.608079
57.2236776	-.0375272075	.0276968923	.0466412816	2.50578465	143.570936
65.7933236	-.0330884779	.0247128768	.0412985915	2.50009615	143.245009
75.6463342	-.0296938247	.021908753	.0369014456	2.50593127	143.579337
86.9749019	-.0271044597	.0193302736	.0332913084	2.52207784	144.504467
100.000002	-.0251332985	.0169946678	.0303397665	2.5470319	145.93423

Figure 10

Figure 10 (Cont.)

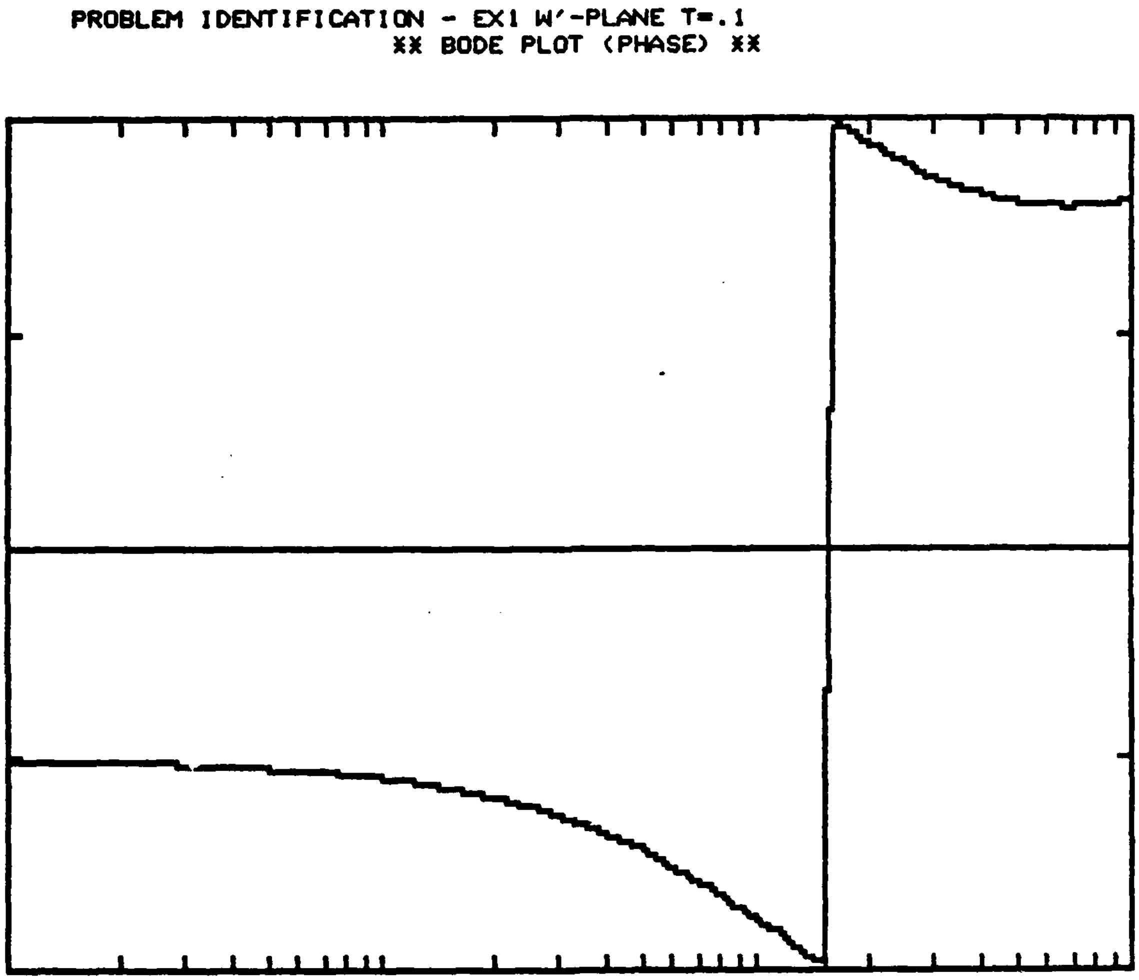
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.1
** BODE PLOT (AMPLITUDE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-40 DECIBELS

Figure 10

Figure 10 (Cont.)



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 100 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 180 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Figure 10

```
ROOT LOCUS
PROBLEM IDENTIFICATION - EX1 S-PLANE
**********************************************

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  1




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  0
  10
  1


OPEN-LOOP POLES
       REAL PART              IMAGINARY PART

       0                      0
       -10                    0

       MIN GAIN               MAX GAIN
       0                      300

**********************************************
```

Figure 11 s-Plane Root Locus Example

Figure 11 (Cont.)

```
1      GAIN = 0                      ROOTS ARE
                          REAL PART            IMAG. PART
                          -10                  0
                          0                    0
2      GAIN = .492857143             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.0495310467         0
                          -9.95046896          0
3      GAIN = 1.05964286             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.107111575          0
                          -9.89288843          0
4      GAIN = 1.71144643             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.174178456          0
                          -9.82582155          0
5      GAIN = 2.46102054             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.252476491          0
                          -9.74752351          0
6      GAIN = 3.32303076             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.344146776          0
                          -9.65585323          0
7      GAIN = 4.31434252             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.451851202          0
                          -9.5481488           0
8      GAIN = 5.45435104             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.578953861          0
                          -9.42104614          0
9      GAIN = 6.76536084             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.729796355          0
                          -9.27020365          0
10     GAIN = 8.27302211             ROOTS ARE
                          REAL PART            IMAG. PART
                          -.910137178          0
                          -9.08986283          0
11     GAIN = 10.0068326             ROOTS ARE
                          REAL PART            IMAG. PART
                          -1.12789883          0
                          -8.87210117          0
12     GAIN = 12.0007146             ROOTS ARE
                          REAL PART            IMAG. PART
                          -1.39454782          0
                          -8.60545218          0
13     GAIN = 14.2936789             ROOTS ARE
                          REAL PART            IMAG. PART
                          -1.72794849          0
                          -8.27205151          0
```

Figure 11

Figure 11 (Cont.)

```
14      GAIN = 16.9305879               ROOTS ARE
                              REAL PART          IMAG. PART
                              -2.15932893        0
                              -7.84067107        0
15      GAIN = 19.9630332               ROOTS ARE
                              REAL PART          IMAG. PART
                              -2.75568122        0
                              -7.24431878        0
16      GAIN = 23.4503453               ROOTS ARE
                              REAL PART          IMAG. PART
                              -3.75514874        0
                              -6.24485127        0
17      GAIN = 27.4607543               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -1.56867915
                              -5                 1.56867915
18      GAIN = 32.0727246               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -2.65945945
                              -5                 2.65945945
19      GAIN = 37.3764904               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -3.51802365
                              -5                 3.51802365
20      GAIN = 43.4758212               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -4.29835098
                              -5                 4.29835098
21      GAIN = 50.4900515               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -5.04876732
                              -5                 5.04876732
22      GAIN = 58.5564163               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -5.79279003
                              -5                 5.79279003
23      GAIN = 67.8327359               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -6.54467233
                              -5                 6.54467233
24      GAIN = 78.5005034               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -7.31440383
                              -5                 7.31440383
25      GAIN = 90.7684361               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -8.10977411
                              -5                 8.10977411
26      GAIN = 104.876559               ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -8.93736867
                              -5                 8.93736867
```

Figure 11

Figure 11 (Cont.)

```
27     GAIN = 121.1009                ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -9.80310663
                              -5                 9.80310663
28     GAIN = 139.758892              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -10.7125577
                              -5                 10.7125577
29     GAIN = 161.215582              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -11.6711431
                              -5                 11.6711431
30     GAIN = 185.890777              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -12.6842728
                              -5                 12.6842728
31     GAIN = 214.267251              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -13.7574435
                              -5                 13.7574435
32     GAIN = 246.900195              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -14.8963148
                              -5                 14.8963148
33     GAIN = 284.428082              ROOTS ARE
                              REAL PART          IMAG. PART
                              -5                 -16.1067713
                              -5                 16.1067713
```

Figure 11

Figure 11 (Cont.)

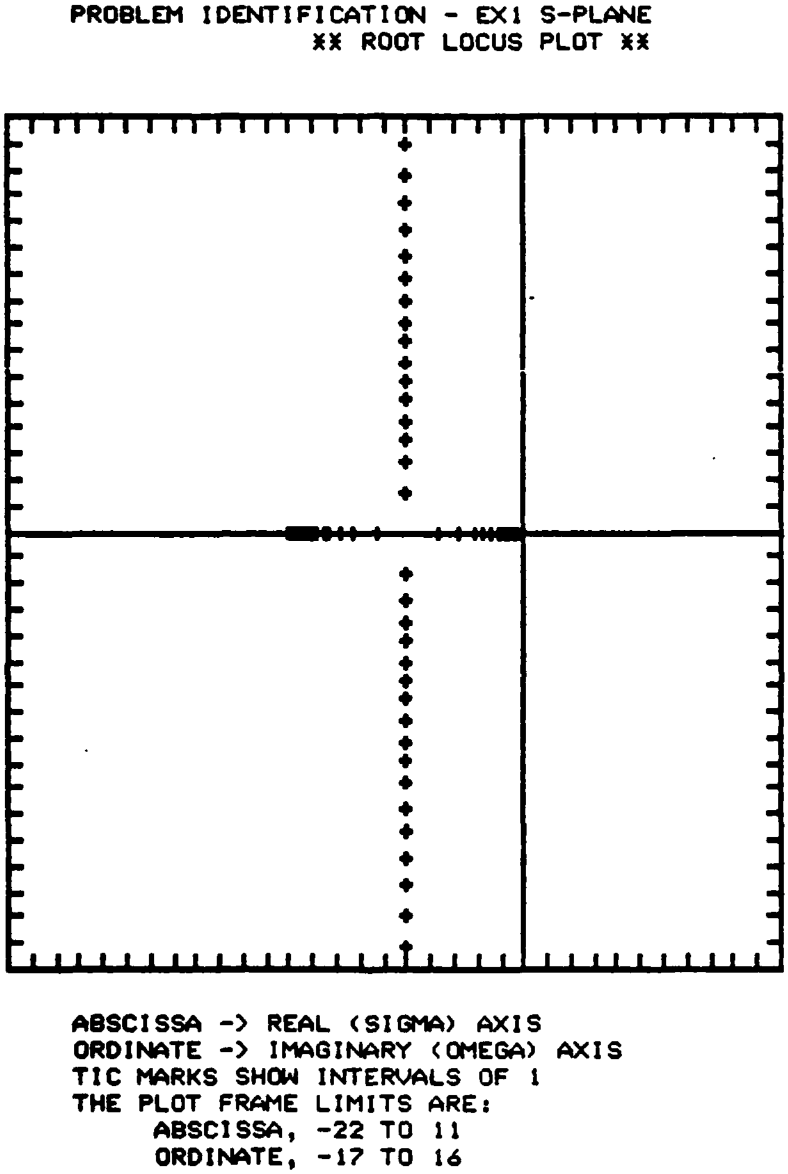


Figure 11

```
ROOT LOCUS
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001
**************************************************

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  -2.405638E+09
  1200819
  1

OPEN-LOOP ZEROS
       REAL PART               IMAGINARY PART

       -1202819                0
       2000                    0




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  0
  9.9999
  1

OPEN-LOOP POLES
       REAL PART               IMAGINARY PART

       0                       0
       -9.9999                 0

       MIN GAIN                MAX GAIN
       0                       -1.2546E-07

**************************************************
```

Figure 12 w'-Plane Root Locus Example, T=.001

Figure 12 (Cont.)

```
1     GAIN = 0                        ROOTS ARE
                                REAL PART           IMAG. PART
                                -9.9999             0
                                0                   0
2     GAIN = -2.06112857E-10          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.0498333602        0
                                -9.94981915         0
3     GAIN = -4.43142643E-10          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.107772379         0
                                -9.89159549         0
4     GAIN = -7.15726896E-10          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.175266637         0
                                -9.82377392         0
5     GAIN = -1.02919879E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.254077448         0
                                -9.74458669         0
6     GAIN = -1.38969146E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.346367788         0
                                -9.65186347         0
7     GAIN = -1.80425804E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.454829223         0
                                -9.54290422         0
8     GAIN = -2.2810096E-09           ROOTS ARE
                                REAL PART           IMAG. PART
                                -.58286724          0
                                -9.41429371         0
9     GAIN = -2.8292739E-09           ROOTS ARE
                                REAL PART           IMAG. PART
                                -.734883239         0
                                -9.26161935         0
10    GAIN = -3.45977784E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -.916725984         0
                                -9.07901949         0
11    GAIN = -4.18485737E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -1.13646232         0
                                -8.85841248         0
12    GAIN = -5.01869884E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -1.40580821         0
                                -8.58806531         0
13    GAIN = -5.97761652E-09          ROOTS ARE
                                REAL PART           IMAG. PART
                                -1.74311005         0
                                -8.24961198         0
```

Figure 12

Figure 12 (Cont.)

```
14     GAIN = -7.08037185E-09              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -2.18070408        0
                                   -7.81069376        0
15     GAIN = -8.34854049E-09              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -2.7890739         0
                                   -7.2008011         0
16     GAIN = -9.80693442E-09              ROOTS ARE
                                   PEAL PART          IMAG. PART
                                   ·3.83271797        0
                                   -6.15540578        0
17     GAIN = -1.14840874E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.9930549         -1.64193793
                                   -4.9930549         1.64193793
18     GAIN = -1.34128134E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.99189689        -2.71059762
                                   -4.99189689        2.71059762
19     GAIN = -1.56308483E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.99056517        -3.56320396
                                   -4.99056517        3.56320396
20     GAIN = -1.81815884E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.98903369        -4.34141261
                                   -4.98903369        4.34141261
21     GAIN = -2.11149395E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.9872725         -5.0913667
                                   -4.9872725         5.0913667
22     GAIN = -2.44882933E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.98524712        -5.83586167
                                   -4.98524712        5.83586167
23     GAIN = -2.83676501E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.98291794        -6.58884116
                                   -4.98291794        6.58884116
24     GAIN = -3.28289105E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.98023937        -7.36014218
                                   -4.98023937        7.36014218
25     GAIN = -3.79593599E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.97715903        -8.15747327
                                   -4.97715903        8.15747327
26     GAIN = -4.38593767E-08              ROOTS ARE
                                   REAL PART          IMAG. PART
                                   -4.97361663        -8.9873759
                                   -4.97361663        8.9873759
```

Figure 12

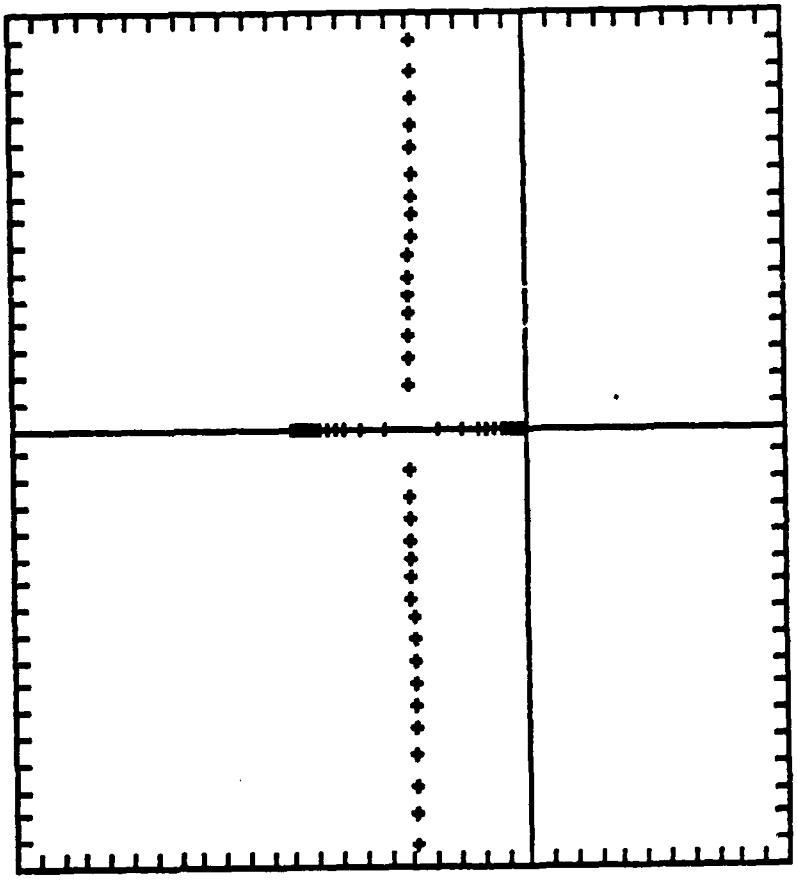
Figure 12 (Cont.)

27	GAIN = -5.06443961E-08	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.96954288	-9.85574623
		-4.96954288	9.85574623
28	GAIN = -5.84471684E-08	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.96485806	-10.7681438
		-4.96485806	10.7681438
29	GAIN = -6.74203565E-08	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.95947052	-11.7299887
		-4.95947052	11.7299887
30	GAIN = -7.77395228E-08	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.95327484	-12.746695
		-4.95327484	12.746695
31	GAIN = -8.96065641E-08	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.94614981	-13.8237686
		-4.94614981	13.8237686
32	GAIN = -1.03253662E-07	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.93795603	-14.9668817
		-4.93795603	14.9668817
33	GAIN = -1.18947824E-07	ROOTS ARE	
		REAL PART	IMAG. PART
		-4.92853319	-16.1819344
		-4.92853319	16.1819344

Figure 12

Figure 12 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.001
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -22 TO 11
ORDINATE, -17 TO 16

Figure 12

```
ROOT LOCUS
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01
**********************************************

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  -2400400.84
  11802.0042
  1

OPEN-LOOP ZEROS
       REAL PART             IMAGINARY PART

       200                   0
       -12002.0042           0




DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

  0
  9.99167498
  1

OPEN-LOOP POLES
       REAL PART             IMAGINARY PART

       0                     0
       -9.99167498           0

       MIN GAIN              MAX GAIN
       0                     -1.25E-04

**********************************************
```

Figure 13 w'-Plane Root Locus Example, T=.01

Figure 13 (Cont.)

```
1     GAIN = 0                      ROOTS ARE
                                REAL PART        IMAG. PART
                                -9.99167498      0
                                0                0
2     GAIN = -2.05357143E-07        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.0495932003     0
                                -9.93966022      0
3     GAIN = -4.41517857E-07        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.107278051      0
                                -9.87919055      0
4     GAIN = -7.13102678E-07        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.174510775      0
                                -9.80875529      0
5     GAIN = -1.02542522E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.253064237      0
                                -9.72651892      0
6     GAIN = -1.38459615E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.345120662      0
                                -9.63022713      0
7     GAIN = -1.79764271E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.453403097      0
                                -9.51707403      0
8     GAIN = -2.27264626E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.581368364      0
                                -9.38350749      0
9     GAIN = -2.81890035E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.73350241       0
                                -9.22493198      0
10    GAIN = -3.44709254E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -.915796521      0
                                -9.03523017      0
11    GAIN = -4.16951357E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -1.1365675       0
                                -8.80594033      0
12    GAIN = -5.00029774E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -1.40799799      0
                                -8.52471313      0
13    GAIN = -5.95569955E-06        ROOTS ARE
                                REAL PART        IMAG. PART
                                -1.74939626      0
                                -8.17204862      0
```

Figure 13

Figure 13 (Cont.)

```
14     GAIN = -7.05441162E-06            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -2.1954399          0
                                  -7.71304879         0
15     GAIN = -8.31793051E-06            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -2.82447443         0
                                  -7.06911459         0
16     GAIN = -9.77097722E-06            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -3.97303464         0
                                  -5.90341974         0
17     GAIN = -1.1441981E-05             ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.92837473         -1.78235168
                                  -4.92837473         1.78235168
18     GAIN = -1.33636352E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.91704436         -2.8109046
                                  -4.91704436         2.8109046
19     GAIN = -1.55735377E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.90401439         -3.65156925
                                  -4.90401439         3.65156925
20     GAIN = -1.81149255E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.88902985         -4.42507143
                                  -4.88902985         4.42507143
21     GAIN = -2.10375214E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.87179752         -5.1735032
                                  -4.87179752         5.1735032
22     GAIN = -2.43985068E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.85198023         -5.91826942
                                  -4.85198023         5.91826942
23     GAIN = -2.82636399E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.82919018         -6.67269846
                                  -4.82919018         6.67269846
24     GAIN = -3.2708543E-05             ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.80298141         -7.4463113
                                  -4.80298141         7.4463113
25     GAIN = -3.78201816E-05            ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.77284103         -8.24663677
                                  -4.77284103         8.24663677
26     GAIN = -4.3698566E-05             ROOTS ARE
                                  REAL PART           IMAG. PART
                                  -4.73817921         -9.08010553
                                  -4.73817921         9.08010553
```

Figure 13

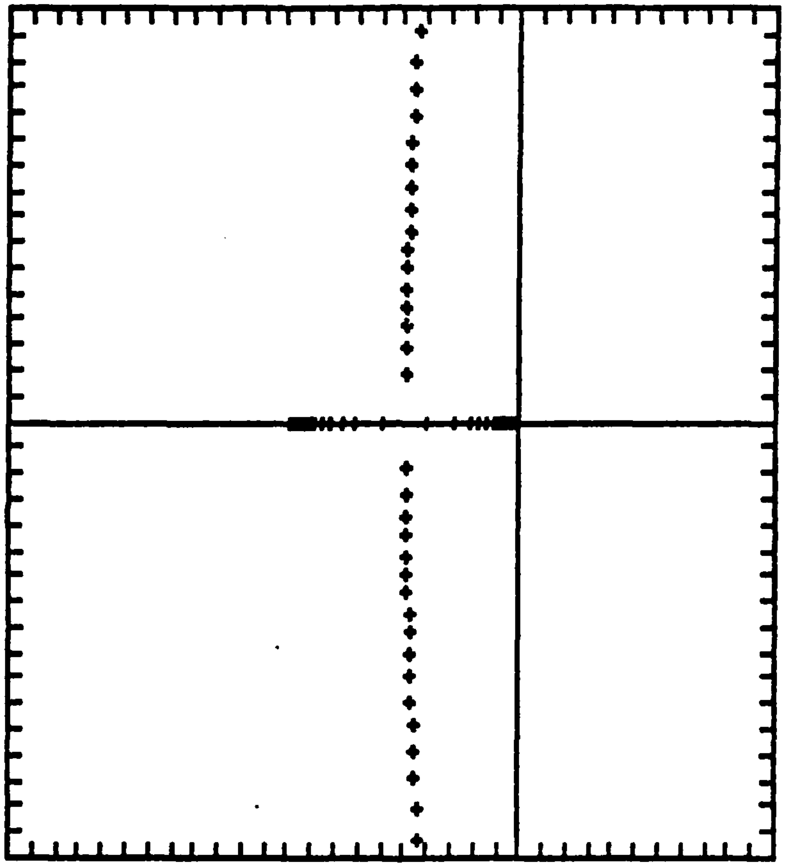
Figure 13 (Cont.)

```
27     GAIN = -5.04587081E-05           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.69831762        -9.95253982
                                  -4.69831762        9.95253982
28     GAIN = -5.82328715E-05           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.65247613        -10.8694452
                                  -4.65247613        10.8694452
29     GAIN = -6.71731593E-05           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.59975752        -11.8361975
                                  -4.59975752        11.8361975
30     GAIN = -7.74544903E-05           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.53912996        -12.8581696
                                  -4.53912996        12.8581696
31     GAIN = -8.9278021E-05            ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.46940673        -13.940824
                                  -4.46940673        13.940824
32     GAIN = -1.02875081E-04           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.38922296        -15.0897833
                                  -4.38922296        15.0897833
33     GAIN = -1.18511701E-04           ROOTS ARE
                                  REAL PART          IMAG. PART
                                  -4.29700894        -16.3108865
                                  -4.29700894        16.3108865
```

Figure 13

Figure 13 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.01
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -22 TO 11
ORDINATE, -17 TO 16

Figure 13

ROOT LOCUS
PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.1
****

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

-2439.71742
101.985871
1

OPEN-LOOP ZEROS

REAL PART	IMAGINARY PART
-121.985871	0
20	0

DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

0
9.24234314
1

OPEN-LOOP POLES

REAL PART	IMAGINARY PART
0	0
-9.24234314	0

MIN GAIN	MAX GAIN
0	-.113648529

****

Figure 14 w'-Plane Root Locus Example, T=.1

Figure 14 (Cont.)

#	GAIN	ROOTS ARE REAL PART	IMAG. PART
1	GAIN = 0	-9.24234314	0
		0	0
2	GAIN = -1.86708298E-04	-.0496547378	0
		-9.17536918	0
3	GAIN = -4.0142284E-04	-.107695744	0
		-9.09740308	0
4	GAIN = -6.48344563E-04	-.175741467	0
		-9.00643291	0
5	GAIN = -9.32304545E-04	-.255807277	0
		-8.89998997	0
6	GAIN = -1.25885852E-03	-.350442074	0
		-8.77500293	0
7	GAIN = -1.6343956E-03	-.462932827	0
		-8.62758256	0
8	GAIN = -2.06626324E-03	-.597624754	0
		-8.45268907	0
9	GAIN = -2.56291102E-03	-.760451059	0
		-8.24358792	0
10	GAIN = -3.13405597E-03	-.959874079	0
		-7.99089184	0
11	GAIN = -3.79087267E-03	-1.20872442	0
		-7.68070196	0
12	GAIN = -4.54621186E-03	-1.52831341	0
		-7.29047243	0
13	GAIN = -5.41485194E-03	-1.95977076	0
		-6.77764581	0

Figure 14

Figure 14 (Cont.)

```
14      GAIN = -6.41378803E-03          ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -2.61028387      0
                                 -6.03338216      0
15      GAIN = -7.56256453E-03          ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -4.26780979      -.613935262
                                 -4.26780979      .613935262
16      GAIN = -8.8836575E-03           ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -4.20552827      -2.04485029
                                 -4.20552827      2.04485029
17      GAIN = -.0104029144             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -4.13369894      -2.92566367
                                 -4.13369894      2.92566367
18      GAIN = -.0121500599             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -4.05082208      -3.68756611
                                 -4.05082208      3.68756611
19      GAIN = -.0141592772             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.95515054      -4.40426744
                                 -3.95515054      4.40426744
20      GAIN = -.016469877              ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.84464502      -5.10621413
                                 -3.84464502      5.10621413
21      GAIN = -.0191270669             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.71692006      -5.8102587
                                 -3.71692006      5.8102587
22      GAIN = -.0221828352             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.56917818      -6.52752509
                                 -3.56917818      6.52752509
23      GAIN = -.0256969688             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.39812932      -7.26633187
                                 -3.39812932      7.26633187
24      GAIN = -.0297382224             ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -3.1998914       -8.03350963
                                 -3.1998914       8.03350963
25      GAIN = -.034385664              ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -2.96986645      -8.83507658
                                 -2.96986645      8.83507658
26      GAIN = -.039730222              ROOTS ARE
                                 REAL PART        IMAG. PART
                                 -2.70258524      -9.67661888
                                 -2.70258524      9.67661888
```

Figure 14

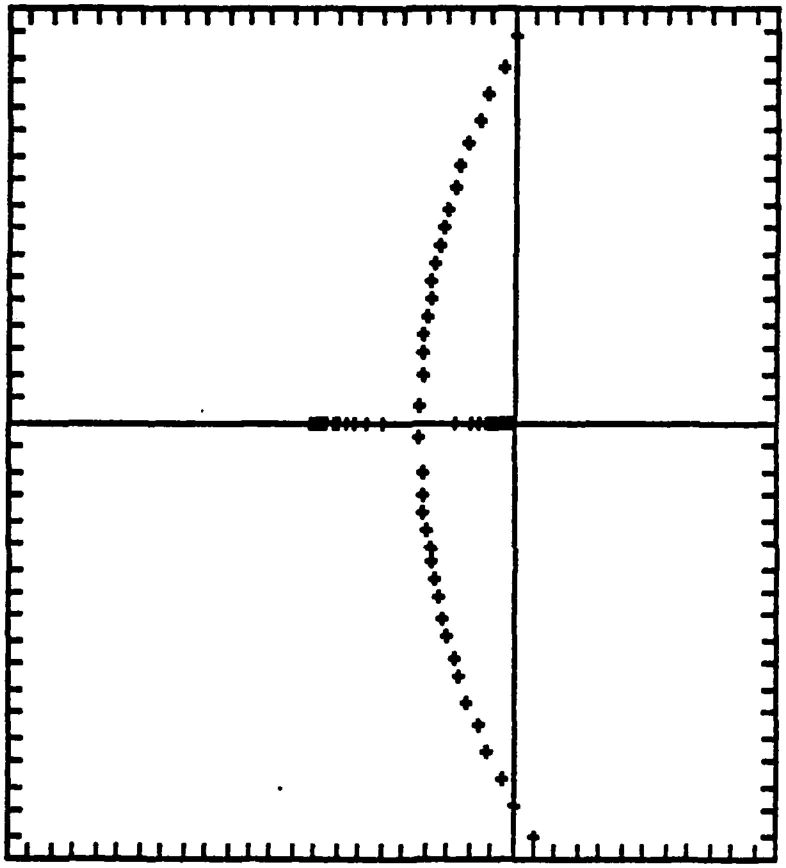
Figure 14 (Cont.)

27	GAIN = -.0458764635	ROOTS ARE	
		REAL PART	IMAG. PART
		-2.39151005	-10.5635188
		-2.39151005	10.5635188
28	GAIN = -.0529446414	ROOTS ARE	
		REAL PART	IMAG. PART
		-2.02878202	-11.5010951
		-2.02878202	11.5010951
29	GAIN = -.0610730459	ROOTS ARE	
		REAL PART	IMAG. PART
		-1.60489341	-12.4946847
		-1.60489341	12.4946847
30	GAIN = -.070420711	ROOTS ARE	
		REAL PART	IMAG. PART
		-1.10825704	-13.549675
		-1.10825704	13.549675
31	GAIN = -.081170526	ROOTS ARE	
		REAL PART	IMAG. PART
		-.524632901	-14.6714825
		-.524632901	14.6714825
32	GAIN = -.0935328132	ROOTS ARE	
		REAL PART	IMAG. PART
		.163647556	-15.8654581
		.163647556	15.8654581
33	GAIN = -.107749443	ROOTS ARE	
		REAL PART	IMAG. PART
		.97874845	-17.1366797
		.97874845	17.1366797

Figure 14

Figure 14 (Cont.)

PROBLEM IDENTIFICATION - EX1 W'-PLANE T=.1
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -23 TO 12
ORDINATE, -18 TO 17

Figure 14

C. ROOT LOCUS TEMPLATES IN w'-PLANE

The s-plane consists of an abscissa representing the real portion of the complex variable s and an ordinate representing the imaginary part where

$$s = \sigma + j\omega$$

The w'-plane approaches the s-plane as the sampling period T approaches zero. (See [Ref. 11] for a proof.) The w'-plane, like the s-plane, consists of an abscissa representing the real part and an ordinate representing the imaginary part where

$$w' = u' + j\nu'$$

The s and w' planes are related as shown below.

$$u' = 2/T \tanh(\sigma T/2)$$

$$\nu' = 2/T \tan(\omega' T/2)$$

$$\sigma = 2/T \tanh^{-1}(u'T/2) \quad \text{where } -1 < u'T/2 < 1$$

$$\omega_d = 2/5 \tan^{-1}(\nu'T/2) \quad \text{where } -1 < \nu'T/2 < 1$$

In the s-plane, σ represents damping. Therefore a line of constant damping in the s-plane is a vertical line parallel to the imaginary axis and passing through the proper value of σ on the real axis. Since the real axis in the w'-plane represents u' which is not the damping but only related to σ as shown above, it is not possible to easily determine the damping of a particular root by simply observing the position of the root in the plane. This is also true of the damped natural frequency, the natural frequency, and the damping ratio. Because of this it is helpful to have templates of constant parameters in the w'-plane to make interpretation of the characteristics of a root by its location easier. The following sections give some insight into the nature of these templates in the w'-plane.

1. Constant Damping

The constant damping templates are shown in Figure 15. There are three templates, one of each of three values of sampling period, .001 seconds, .01 seconds, and .1 seconds. Each template shows lines of constant damping for the values 25, 50, 75, and 100. These templates are created by using the above relationship between σ and u' to find the constant value of u' that corresponds to the constant value of σ and then plotting the constant u' value. From the templates it is seen that for a period of .001 there is practically no difference from the s-plane. For a period of .01 seconds, some distortion is noticed as the values of damping approach the reciprocal of the value of the period. This distortion is in the form of u' becoming smaller than the value of damping. And for a period of .1 seconds gross distortion is seen. Figure 15 shows an extra template for a period of .1 seconds but for values of damping of 1, 2, 3, and 4. In this case only little distortion is present because the values of damping are small compared to the reciprocal of the period.

2. Constant Damped Natural Frequency

Figure 16 shows the templates for constant damped natural frequency for values of damped natural frequency of 25, 50, 75, and 100. The results here are similar to the constant damping case. As the period increases, the distortion between the value of ω_d and the value of ν' increases. In this case the value of ν' becomes greater than the value of damped natural frequency as it distorts. In the case where the period is .1 seconds the distortion is so great that the template is useless and negative values of ν' are produced by the tangent function in the relationship.

3. Constant Natural Frequency

Templates for constant natural frequency in the w'-plane are shown in Figure 17. In the s-plane constant natural frequency ω_n plots as a circle with its center at the origin where

$$\omega_n = \mathrm{sqrt}(\sigma^2 + \omega_d^2)$$

To plot constant natural frequency in the w'-plane, substitute

$$u' = 2/T \tanh(\sigma T/2)$$
$$\nu' = 2/T \tan(\omega_d T/2)$$

into the above equaticn and solve for ν' to get

$$\nu' = 2/T \tan \left[T/2 \{\omega_n^2 - [2/T \tanh^{-1} (u'T/2)]^2\}^{\frac{1}{2}} \right]$$

For a constant value of natural frequency and period this equation is used to plot u' vs ν'. In the w'-plane for a period of .001 seconds and a value of natural frequency of 25 it plots very close to a perfect circle. As the values of natural frequency increase they form concentric ellipses which are elongated along the imaginary axis and contracted along the real axis. This effect becomes more dramatic as the period increases. An extra template for a period of .05 seconds is added to better show the trend since the change from .01 seconds to .1 seconds is so great.

4. Constant Damping Ratio

The damping ratio, ζ, can be defined as

$$\zeta = \left| \sigma / \omega_n \right| = \cos\theta$$

where $|\,|$ represents the absolute value and θ is the angle formed by the negative real axis and a line joining the origin and the root in a root locus plot. In the s-plane a plot of constant damping ratio is a line radially out from the origin forming the correct angle θ with the negative real axis. To plot constant damping ratios in the w'-plane, substitute the proper relations into the above equation to get

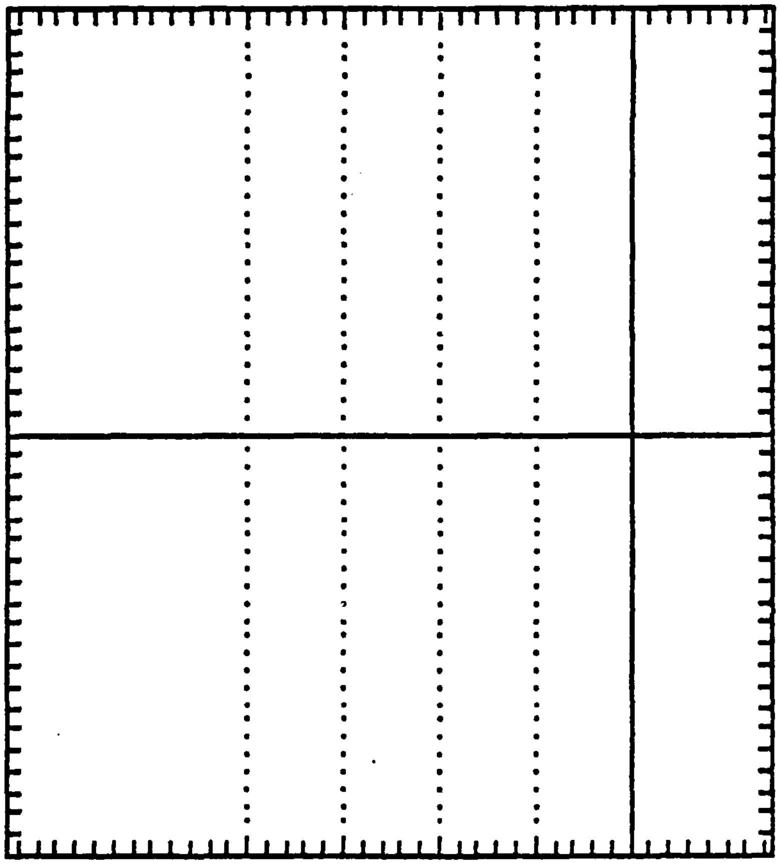
$$\zeta = \frac{\tanh^{-1}(u'T/2)}{\{[\tanh^{-1}(u'T/2)]^2 + [\tan^{-1}(\nu' T/2)]^2\}^{\frac{1}{2}}}$$

Solving for u' gives

$$u' = 2/T \tanh\left[\frac{\tan^{-1}(\nu' T/2)\zeta}{\mathrm{sqrt}(1-\zeta^{2})}\right]$$

This equation is used to plot constant damping ratio for a given sampling period. Figure 18 shows templates for constant damping ratio for values of damping ratio of .1, .5, .707, and .9. From these templates it can be seen that there is a region near the origin that can be interpreted the same as the s-plane for a close approximation and that the size of this region depends on the value of the sampling period. Beyond this region a template makes the interpretation easier.

CONSTANT DAMPING W'-PLANE T=.001
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -163 TO 37
ORDINATE, -100 TO 100

Figure 15 Constant Damping Templates

Figure 15 (Cont.)

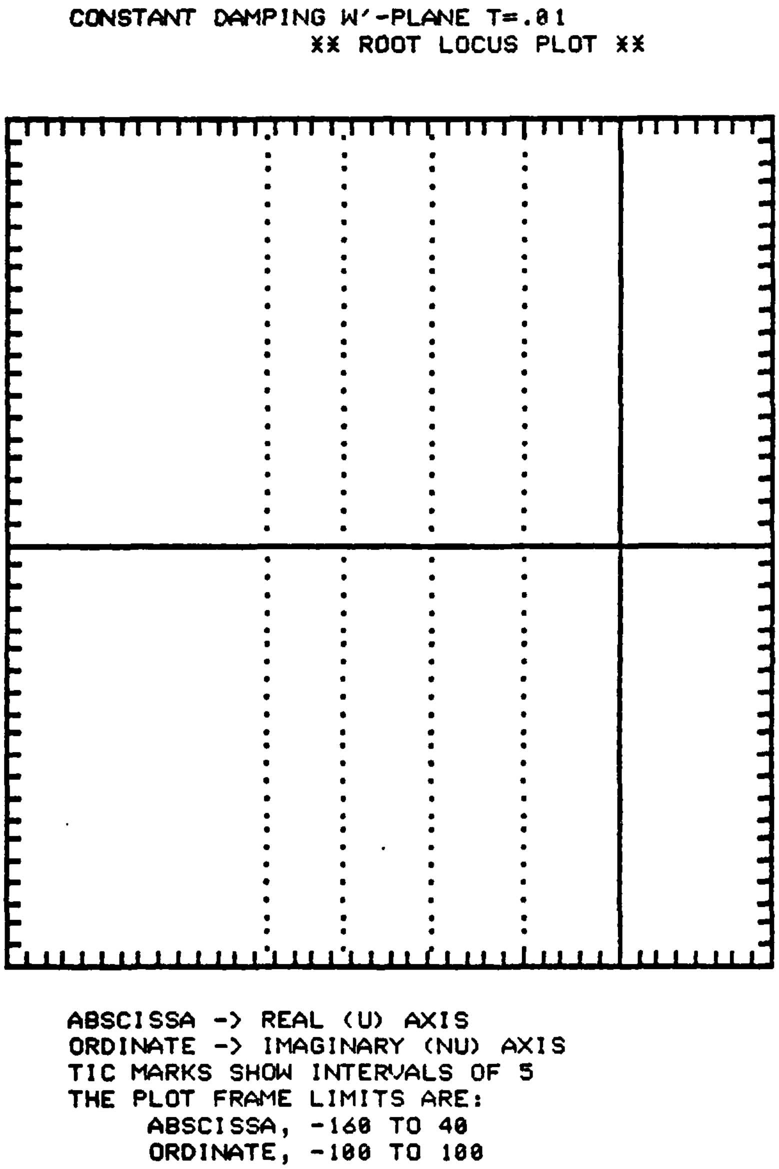
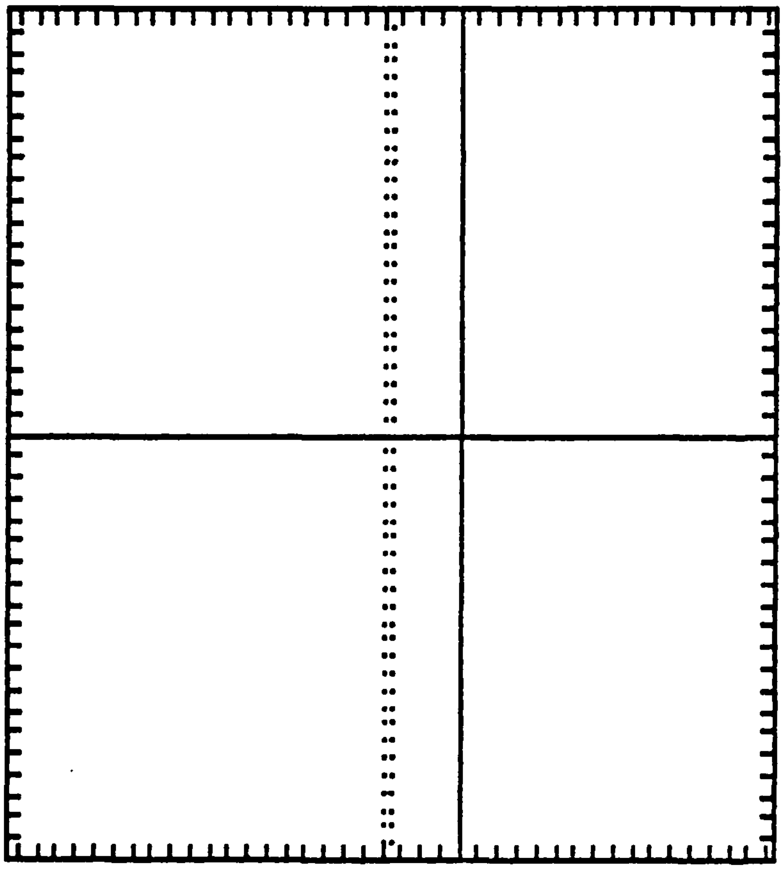


Figure 15

Figure 15 (Cont.)

CONSTANT DAMPING W'-PLANE T=.1
** ROOT LOCUS PLOT **

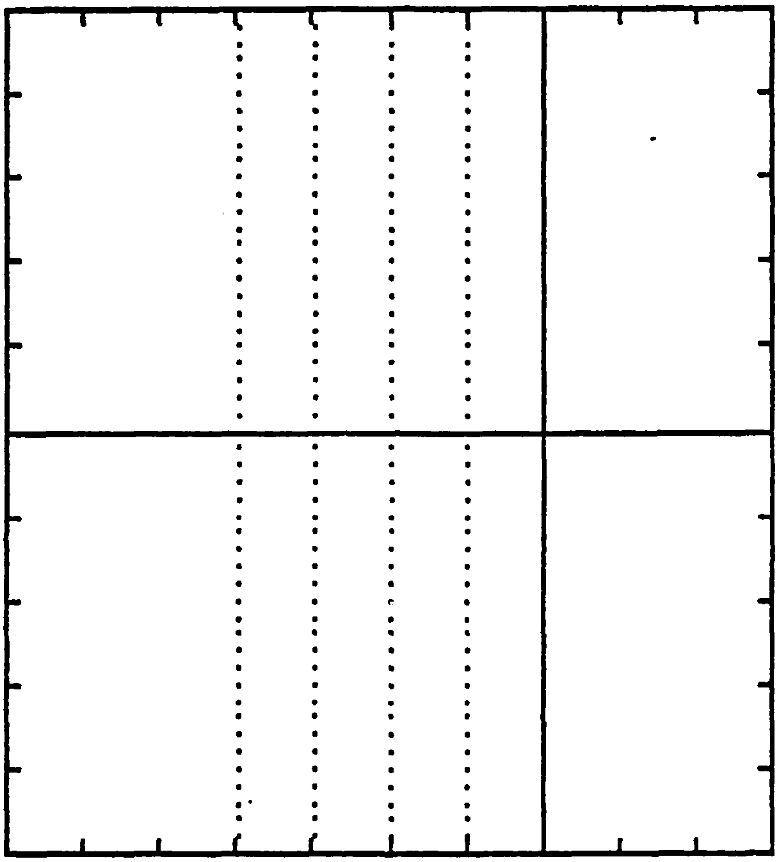


ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -118 TO 82
ORDINATE, -100 TO 100

Figure 15

Figure 15

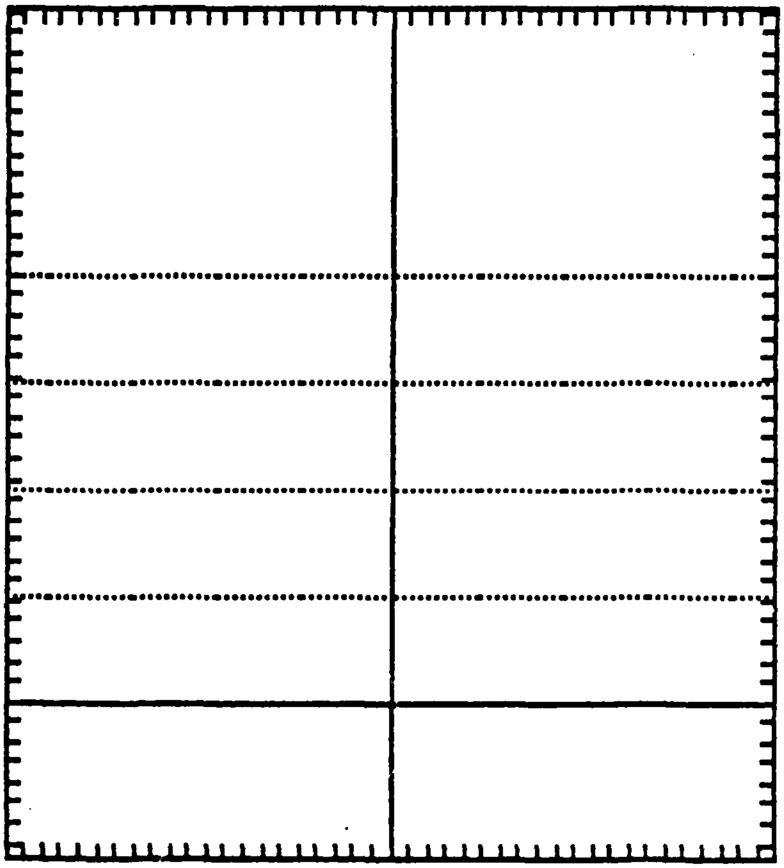
CONSTANT DAMPING W'-PLANE T=.1
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -7 TO 3
ORDINATE, -5 TO 5

Figure 15

CONSTANT DAMPED NATURAL FREQ W'-PLANE T=.001
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -100 TO 100
ORDINATE, -37 TO 163

Figure 16 Constant Damped Natural Frequency Templates

Figure 16 (Cont.)

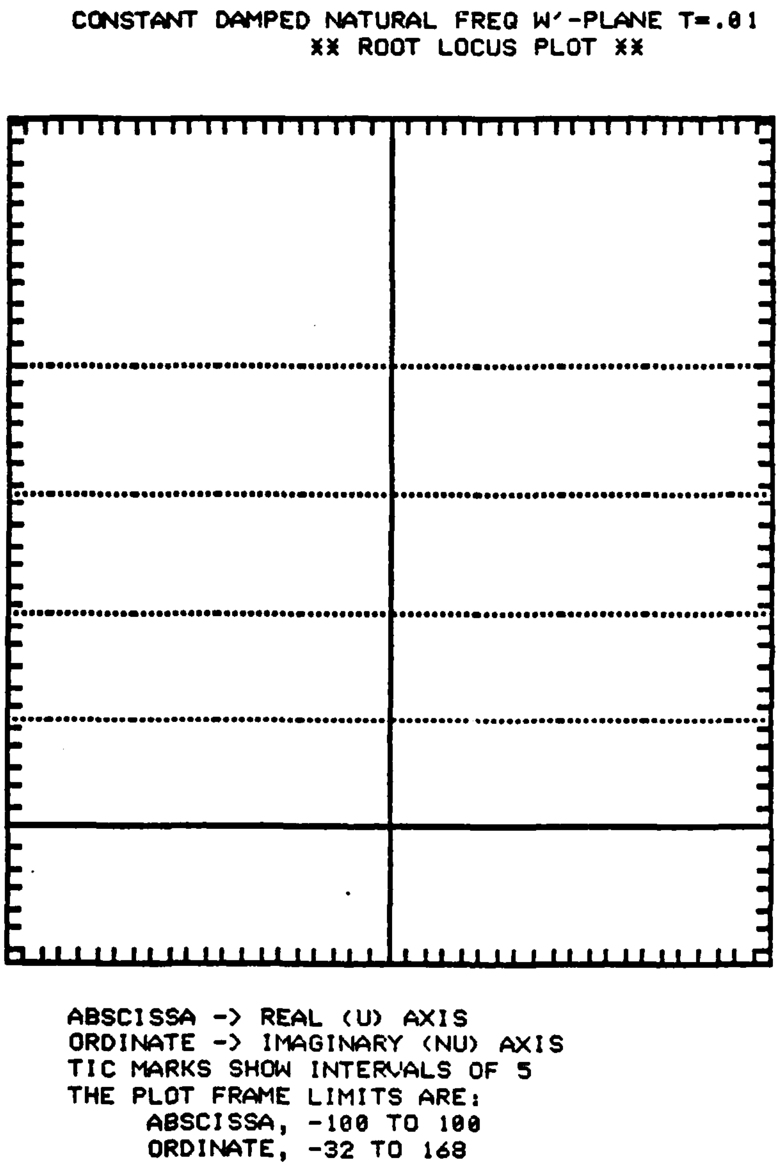
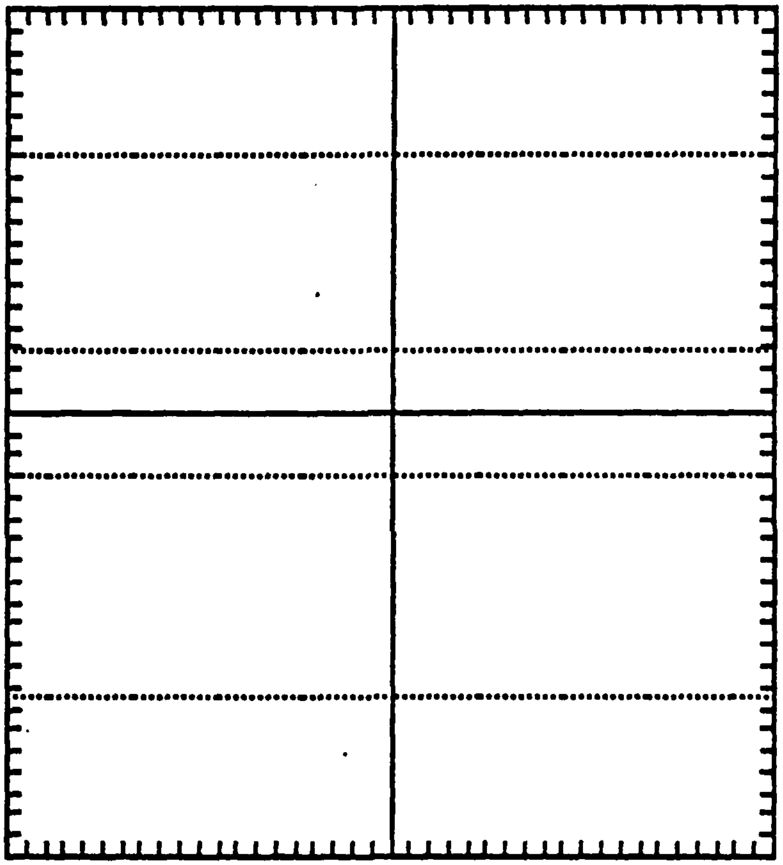


Figure 16

Figure 16 (Cont.)

CONSTANT DAMPED NATURAL FREQ W'-PLANE T=.1
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -100 TO 100
ORDINATE, -105 TO 95

Figure 16

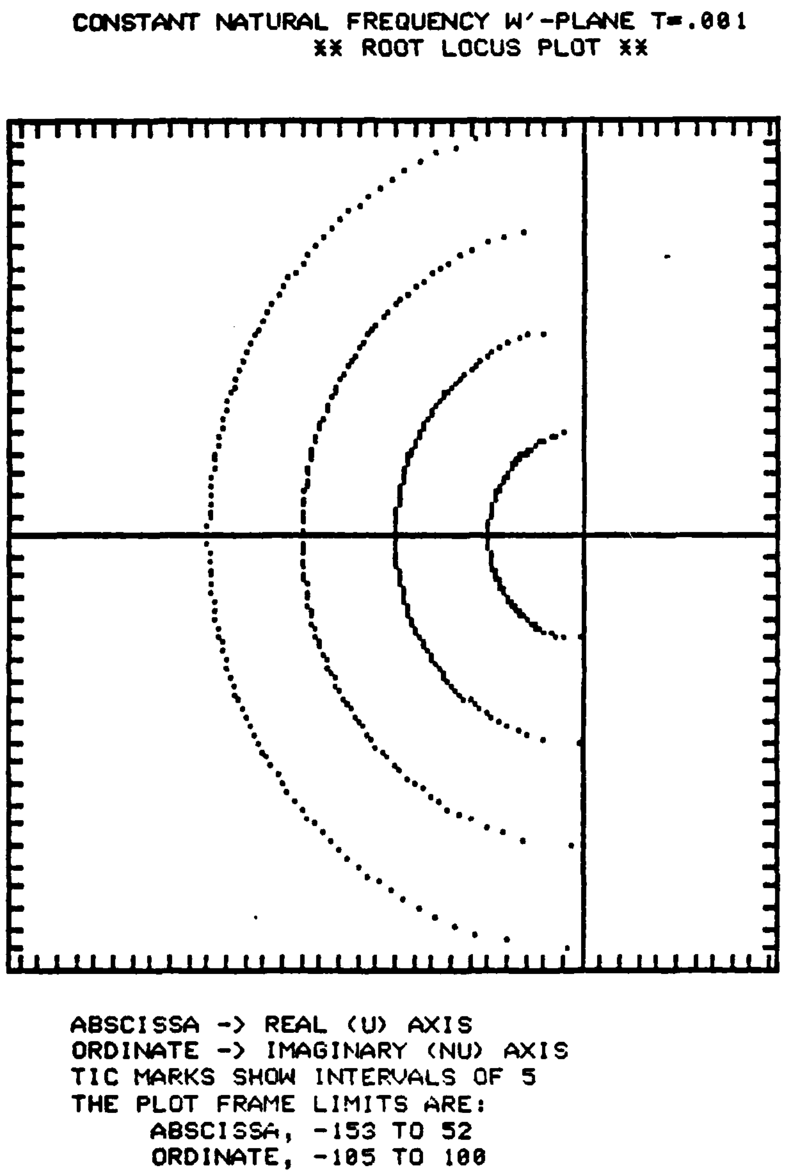
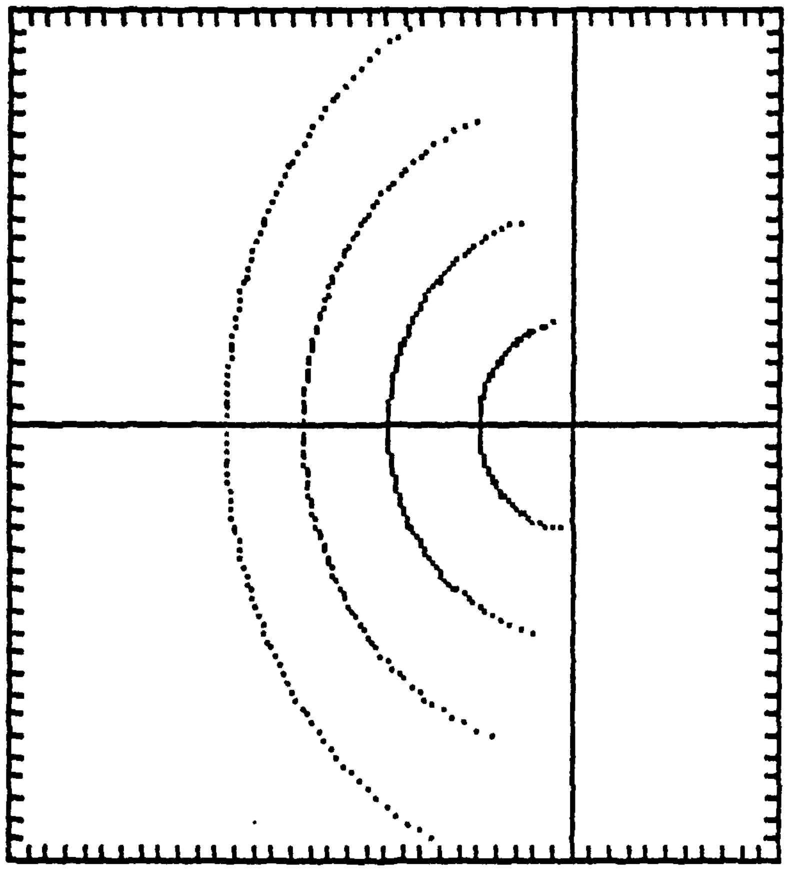


Figure 17 Constant Natural Frequency Templates

Figure 17 (Cont.)

CONSTANT NATURAL FREQUENCY W'-PLANE T=.01
** ROOT LOCUS PLOT **

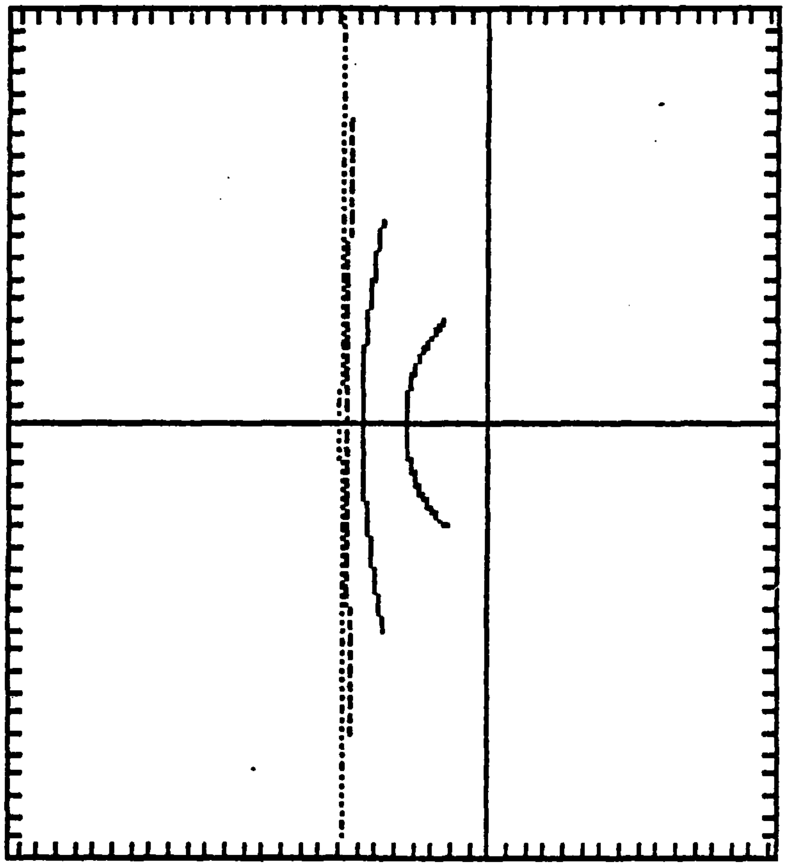


ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -150 TO 55
ORDINATE, -105 TO 100

Figure 17

Figure 17 (Cont.)

CONSTANT NATURAL FREQUENCY W'-PLANE T=.05
** ROOT LOCUS PLOT **

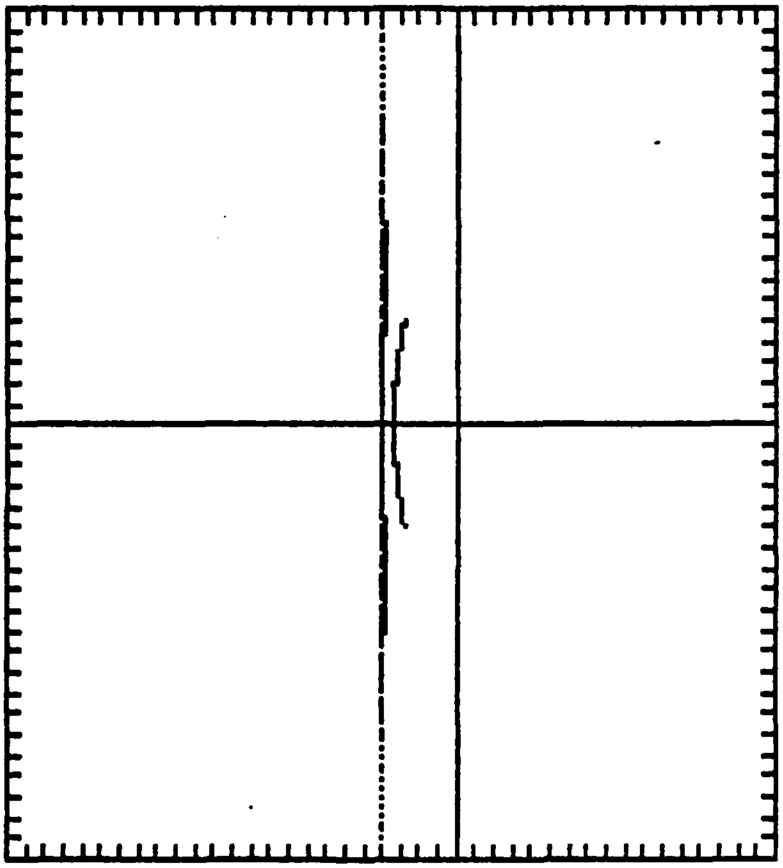


ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -128 TO 77
ORDINATE, -105 TO 100

Figure 17

Figure 17 (Cont.)

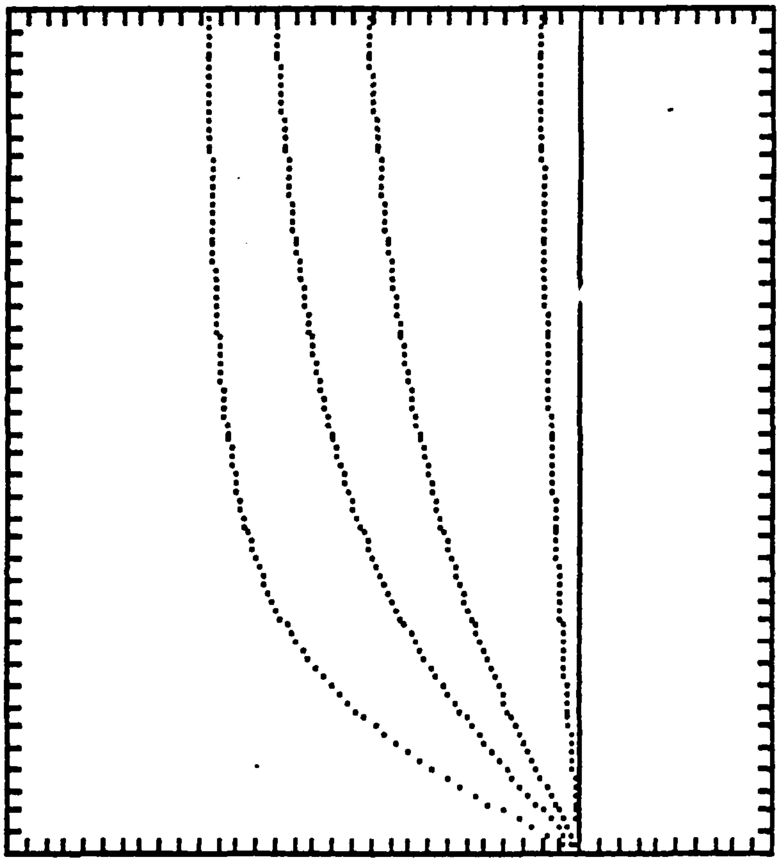
CONSTANT NATURAL FREQUENCY W'-PLANE T=.1
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 5
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -120 TO 85
ORDINATE, -105 TO 100

Figure 17

CONSTANT DAMPING RATIO W'-PLANE T=.001
** ROOT LOCUS PLOT **

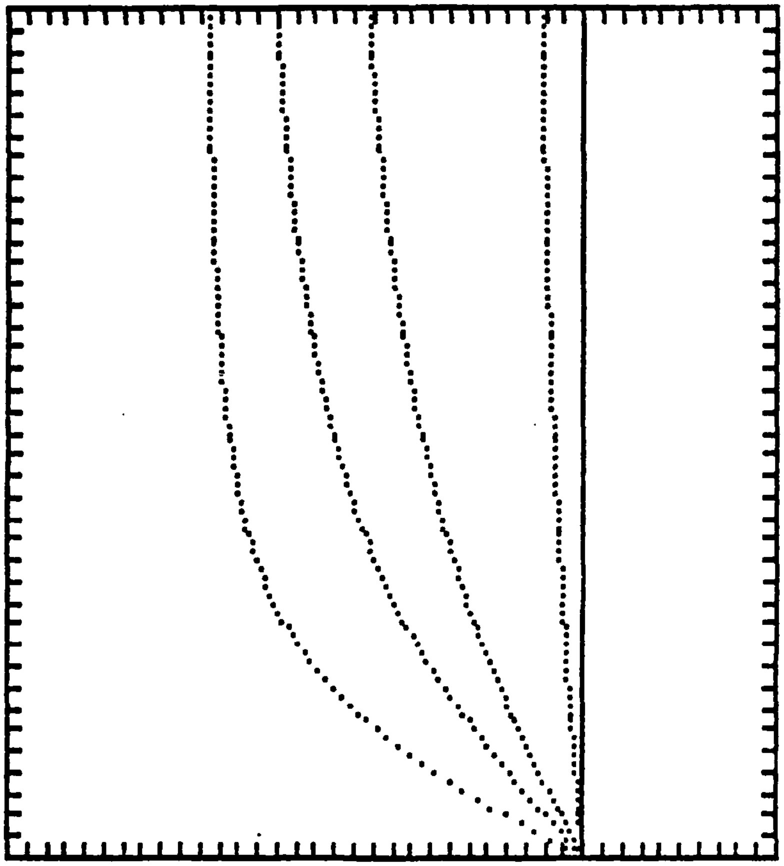


ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 100
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -3000 TO 1000
ORDINATE, 0 TO 4000

Figure 18 Constant Damping Ratio Templates

Figure 18 (Cont.)

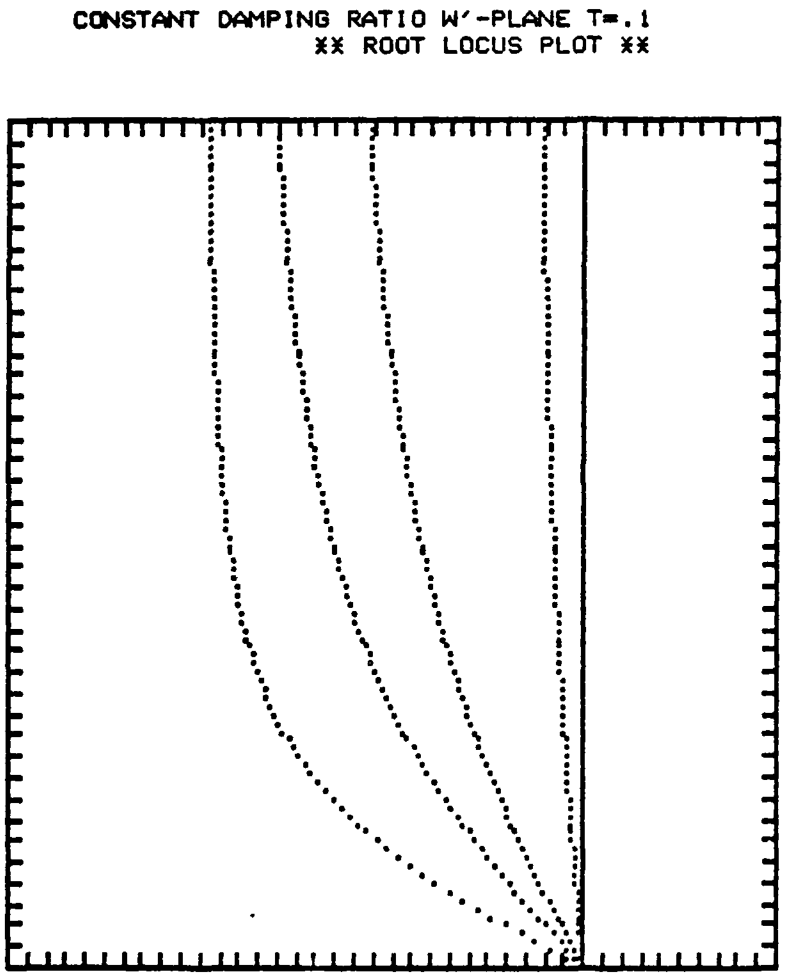
CONSTANT DAMPING RATIO W'-PLANE T=.01
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 10
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -300 TO 100
ORDINATE, 0 TO 400

Figure 18

Figure 18 (Cont.)



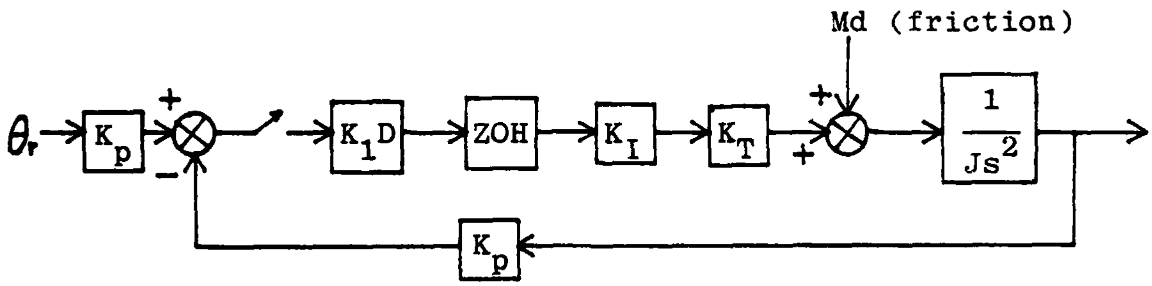
ABSCISSA -> REAL (U) AXIS
ORDINATE -> IMAGINARY (NU) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -30 TO 10
ORDINATE, 0 TO 40

Figure 18

D. COMPENSATION EXAMPLE

To further demonstrate the use of the RTLOC and FRESP programs for analysis and design in the w'-plane, an example from [Ref. 6] is duplicated here.

An angular position servo is described below.

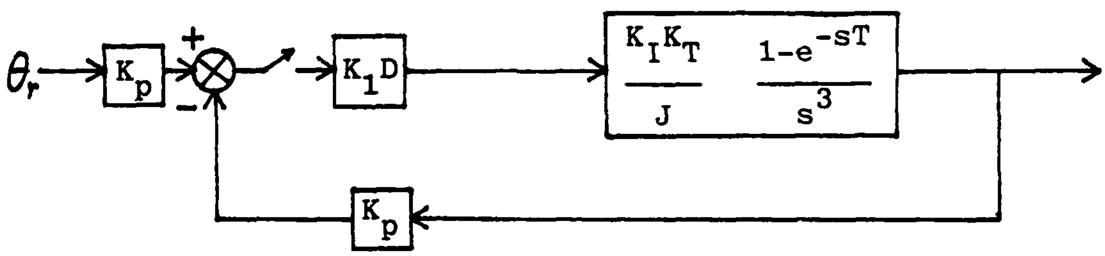


where

K_p = 1.9 (V/rad) $\qquad$ K_I = 1(A/V)

K_T = 0.385 (Nm/A) $\qquad$ J = 3.22×10^3 (Nm. Sec^2)

Neglecting friction (Md) gives



where

$$ZOH = \frac{1-e^{-sT}}{s}$$

$$G_1(s) = \frac{K_I K_T}{J} \quad \frac{1-e^{-sT}}{s^3}$$

Converting to the z domain gives

$$G_1(z) = K_2 T^2 \frac{(z+1)}{(z-1)^2}$$

where

$$K_2 = \frac{K_I K_T}{2J} = 59.8$$

Converting this to the w domain gives

$$G_1(w) = \frac{K_2 T^2}{2} \quad \frac{1-w}{w^2}$$

where

$$w = w'T/2$$

The open loop transfer function is

$$G(w) = K_p K_1 G_1(w)$$

where

$$K_1 = 3.83$$

from the calculation of D C gain. This gives

$$G(w) = 217.58\ T^2\ \frac{1-w}{w^2}$$

For

$$T = 0.01$$

$$G(w) = 0.0218\ \frac{1-w}{w^2} \tag{5}$$

Therefore for the uncompensated system equation 5 above is entered into the FRESP program producing the output shown in Figure 19. From these Bode plots it is seen that the uncompensated system is unstable.

From analog design techniques the final compensation network [Ref. 7] is

$$K_1D(w) = 3.83\ \frac{(w/0.1 + 1)}{(w/2.15 + 1)^2}$$

and the open loop transfer function becomes

$$G(w) = K\ \frac{w+0.1}{(w+2.15)^2}\ \ \frac{w-1}{w^2}$$

For

$$K = -1.017$$

this transfer function was entered into the FRESP program. The resulting Bode plots in Figure 20 show that the compensated system is stable with a gain margin of 10.5 dB and phase margin of 41 degrees.

Figure 21 shows the output from the RTLOC program for the compensated system. This figure includes the normal root locus in addition to an expanded portion of the root locus to obtain more detail. These plots are of w and not w' in this case. Values of w' can be found by the simple relation

$$w' = 2w/T$$

where

$$T = .01 \text{ seconds}$$

for this problem.

On the expanded root locus plot in Figure 21, a root associated with a certain gain may be identified and templates similar to the ones discussed in section V.C. may be used to easily obtain the characteristics associated with the root.

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - KATZ EXAMPLE
*************************************************

GAIN=-.022

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  -1
   1


NUMERATOR ROOTS ARE
       REAL PART             IMAGINARY PART

       1                     0





DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

   0
   0
   1

DENOMINATOR ROOTS ARE
       REAL PART             IMAGINARY PART

       0                     0
       0                     0

*************************************************
```

Figure 19 Uncompensated System Frequency Response

Figure 19 (Cont.)

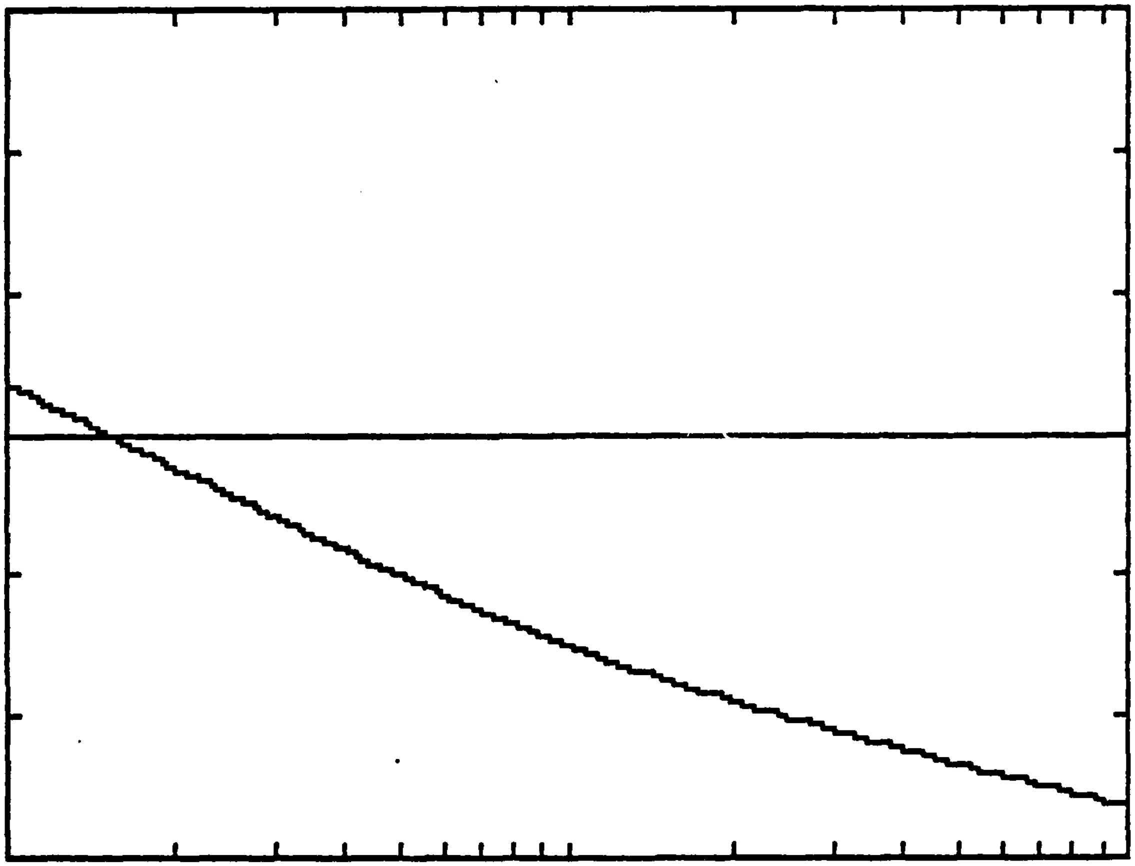
PROBLEM IDENTIFICATION - KATZ EXAMPLE

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.104761575	-2.00455006	.21000866	2.01552805	3.03721188	174.019434
.1149757	-1.66421932	.19134478	1.6751832	3.02711966	173.441242
.126185688	-1.38166411	.174346238	1.39262067	3.01607043	172.808168
.138488637	-1.14708182	.158857798	1.15802958	3.00397938	172.115402
.151991109	-.952327477	.144745309	.963264672	2.99075603	171.357759
.166810054	-.790639001	.131886535	.801563527	2.97630452	170.529748
.183073829	-.656402388	.120170098	.667311733	2.960524	169.625591
.200923301	-.544956794	.109494518	.555847962	2.94330951	168.639273
.220513075	-.452432704	.0997673267	.463302138	2.92455306	167.564607
.242012828	-.375617579	.0909042724	.386461062	2.90414539	166.395333
.26560878	-.311844312	.0828285873	.322656861	2.88197822	165.125248
.291505308	-.258898627	.0754703239	.269674375	2.85794731	163.748377
.319926716	-.214942188	.0687657482	.225674261	2.83195624	162.259198
.351119176	-.17844878	.0626567886	.189129163	2.80392119	160.652908
.385352862	-.148151312	.0570905323	.158770716	2.77377659	158.925748
.422924291	-.122997822	.0520187667	.133545559	2.74148151	157.075376
.464158887	-.102114953	.0473975628	.112578828	2.70702683	155.101268
.509413806	-.0847776276	.0431868939	.0951438593	2.67044244	153.005136
.559081023	-.0703838757	.0393502893	.0806370586	2.63100424	150.791329
.613590733	-.0584339301	.035854518	.0685570613	2.5912399	148.467163
.673415072	-.0485128753	.0326693015	.0584874545	2.54893266	146.043136
.739072211	-.0402762413	.0297670507	.0500824612	2.50512214	143.532977
.81113084	-.0334380431	.027122628	.0430550773	2.46010142	140.953479
.890215095	-.0277608508	.0247131284	.0371672377	2.41421	138.324093
.977009969	-.023047546	.0225176822	.0322216603	2.3678227	135.666296
1.07226724	-.0191344776	.0205172734	.0280550662	2.32133516	133.002755
1.17681197	-.0158857794	.0186945753	.0245325322	2.27514727	130.356383
1.29154968	-.0131886531	.0170338008	.02154277	2.22964625	127.749366
1.41747418	-.0109494515	.0155205649	.0189941681	2.18519088	125.202259
1.55567617	-9.09042702E-03	.0141417607	.0168114621	2.14209859	122.733252
1.70735267	-7.54703221E-03	.0128854456	.0149329302	2.10063635	120.35764
1.87381745	-6.26567872E-03	.0117407381	.01330803	2.06101571	118.087544
2.05651234	-5.20187654E-03	.0106977233	.011895411	2.02339181	115.931852
2.25701976	-4.31868929E-03	9.74736704E-03	.0106612495	1.98786581	113.896362
2.4770764	-3.58545172E-03	8.88143782E-03	9.57785998E-03	1.95448981	111.984057
2.71858829	-2.97670501E-03	8.09243536E-03	8.62254503E-03	1.92327327	110.195481
2.98364729	-2.47131279E-03	7.3735257E-03	7.77664891E-03	1.89419026	108.529146
3.27454922	-2.05172729E-03	6.71848199E-03	7.02478364E-03	1.86718668	106.981955
3.59381373	-1.70338004E-03	6.12163057E-03	6.35420053E-03	1.84218708	105.549582
3.94420613	-1.41417603E-03	5.57780178E-03	5.75428246E-03	1.81910068	104.226829
4.32876137	-1.17407378E-03	5.08228524E-03	5.21613578E-03	1.79782662	103.007915
4.75081026	-9.74736682E-04	4.63078902E-03	4.73226358E-03	1.77825822	101.886727
5.2140084	-8.09243516E-04	4.21940249E-03	4.29630451E-03	1.76028637	100.857016
5.72236778	-6.71848184E-04	3.8445624E-03	3.90282462E-03	1.74380221	99.9125428
6.28029158	-5.57780164E-04	3.50302207E-03	3.5471513E-03	1.72869909	99.047197
6.89261226	-4.63078891E-04	3.19182324E-03	3.22524072E-03	1.71487396	98.2550756
7.56463344	-3.84456231E-04	2.90827046E-03	2.93357183E-03	1.70222942	97.5305389
8.30217587	-3.19182317E-04	2.64990773E-03	2.66906132E-03	1.69066924	96.8682463
9.11162777	-2.64990767E-04	2.41449723E-03	2.42899505E-03	1.68010876	96.2631754
10.0000002	-2.19999992E-04	2.19999995E-03	2.21097261E-03	1.67046503	95.71063

Figure 19

Figure 19 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** BODE PLOT (AMPLITUDE) **

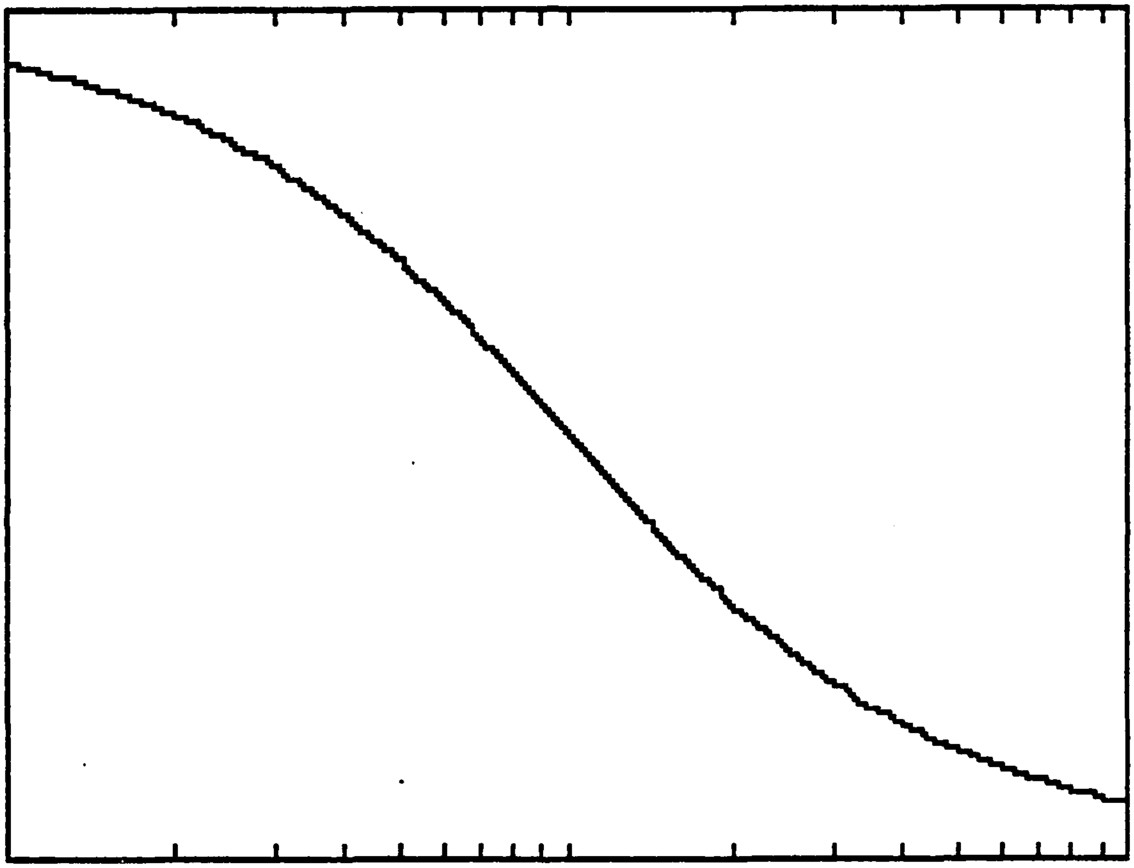


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 10 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-60 DECIBELS

Figure 19

Figure 19 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** BODE PLOT (PHASE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 10 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 180 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = 90 DEGREES

Figure 19

```
FREQUENCY RESPONSE
PROBLEM IDENTIFICATION - KATZ EXAMPLE
*********************************************

GAIN=-1.017

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  -.1
  -.9
  1


NUMERATOR ROOTS ARE
       REAL PART               IMAGINARY PART

       -.1                     0
       1                       0



DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S

  0
  0
  4.6225
  4.3
  1

DENOMINATOR ROOTS ARE
       REAL PART               IMAGINARY PART

       -2.15                   0
       -2.15                   0
       0                       0
       0                       0

*********************************************
```

Figure 20 Compensated System Frequency Response

Figure 20 (Cont.)

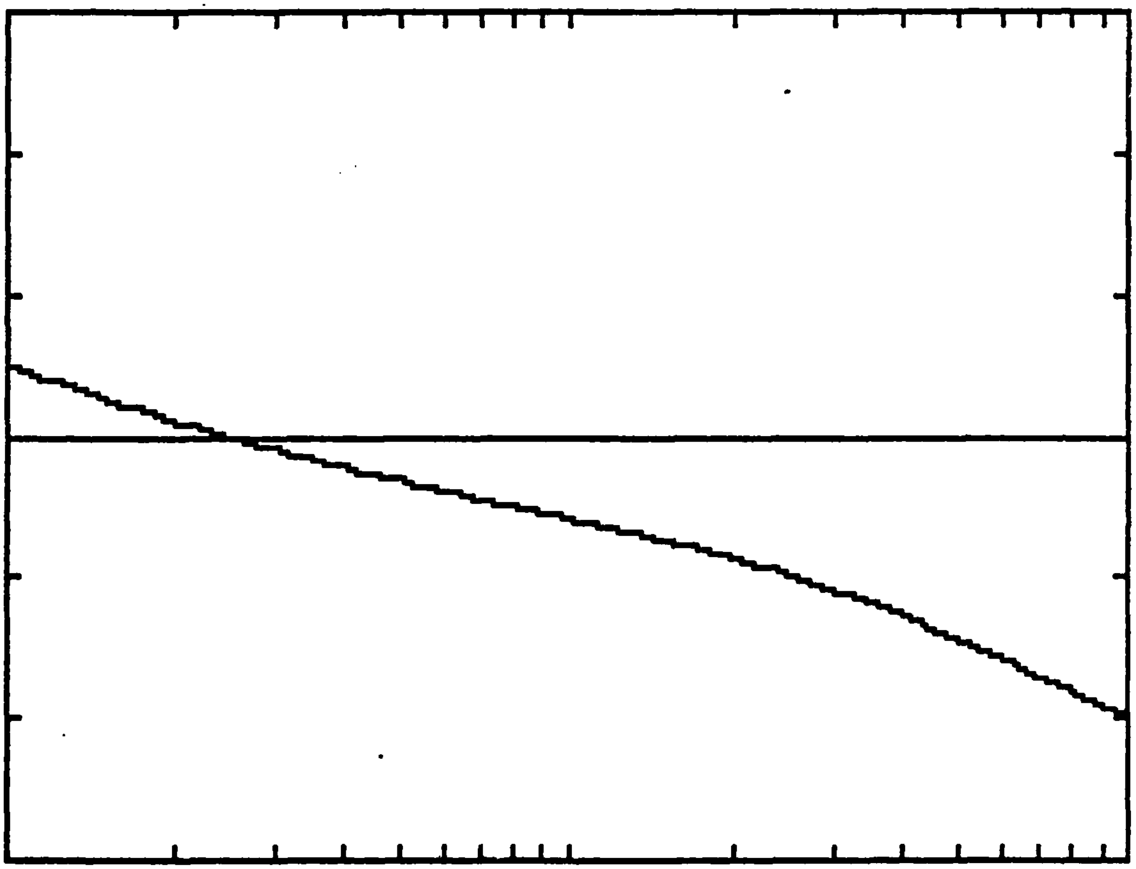
PROBLEM IDENTIFICATION - KATZ EXAMPLE

RADIAN FREQ.	REAL PART	IMAGINARY PART	MAGNITUDE	PHASE(RAD)	PHASE(DEG)
.104761575	-2.39220771	-1.66091537	2.91226675	-2.53470093	-145.227717
.1149757	-2.05136854	-1.50709532	2.54547622	-2.50797001	-143.696148
.126185688	-1.76821783	-1.36634283	2.23461116	-2.48370669	-142.305962
.138488637	-1.5329249	-1.23744316	1.97005693	-2.46245077	-141.088087
.151991109	-1.33732101	-1.11928564	1.74391164	-2.44471727	-140.072032
.166810054	-1.17461586	-1.01085445	1.54969324	-2.43098665	-139.285325
.183073829	-1.03916202	-.911220404	1.38209273	-2.42169891	-138.753176
.200923301	-.926259056	-.819533617	1.23676643	-2.41725149	-138.498358
.220513075	-.831990732	-.735017078	1.11016156	-2.41800032	-138.541263
.242012828	-.7530695	-.656961126	.999370661	-2.42426292	-138.900083
.26560878	-.686823804	-.584718837	.902010563	-2.43632268	-139.591057
.291505308	-.630904245	-.517702353	.816122474	-2.45443313	-140.62871
.319926716	-.583405553	-.455380223	.740089985	-2.47882133	-142.026051
.351119176	-.542701741	-.397275798	.672572108	-2.50968973	-143.794681
.385352862	-.507412466	-.342966767	.612448866	-2.54721596	-145.944776
.422924291	-.476358936	-.292085857	.558777222	-2.59155043	-148.484955
.464158887	-.44852822	-.244322736	.510755483	-2.64281151	-151.421999
.509413806	-.42304507	-.199427017	.46769463	-2.70107883	-154.760472
.559081023	-.399150742	-.157212169	.428995316	-2.76638484	-158.502233
.613590733	-.376188516	-.117559888	.394129591	-2.83870547	-162.645901
.673415072	-.353595756	-.0804242261	.36262655	-2.91795058	-167.186313
.739072211	-.330902379	-.0458343711	.334061632	-3.00395521	-172.114017
.81113084	-.307735349	-.0138945796	.308048867	-3.09647228	-177.414856
.890215095	-.283828366	.0152206099	.284236184	3.08801791	176.930457
.977009969	-.259035071	.0412779193	.262303326	2.98356871	170.945956
1.07226724	-.233343053	.0640054395	.241962139	2.87387962	164.661232
1.17681197	-.20688473	.0831222496	.222958741	2.75955236	158.11076
1.29154968	-.179940328	.0983768522	.20507688	2.64127174	151.333777
1.41747418	-.152928019	.109591363	.188141559	2.51979887	144.373892
1.55567617	-.12637755	.116705736	.172021842	2.39596163	137.278538
1.70735267	-.100886423	.119814167	.15663175	2.27064179	130.098238
1.87381745	-.0770618877	.119185249	.141928356	2.14475823	122.885639
2.05651234	-.0554563234	.115259292	.127906638	2.01924576	115.694301
2.25701976	-.036506795	.108620508	.114591277	1.89503025	108.577274
2.4770764	-.0204899108	.0999475468	.102026215	1.77300134	101.58553
2.71858829	-7.50023915E-03	.0899511678	.0902633158	1.65398518	94.7664037
2.98364729	2.54491874E-03	.0793108303	.0793516504	1.53871942	88.1621603
3.27454922	9.37769306E-03	.0686214773	.0693287529	1.4278337	81.808874
3.59381373	.0148366908	.0583582695	.0602147409	1.32183615	75.7356599
3.94420613	.0178187351	.0488619345	.0520095757	1.22110813	69.9643671
4.32876137	.0192341152	.0403426486	.0446931816	1.12590582	64.5096745
4.75081026	.0194712689	.0328972762	.038227753	1.03636815	59.3795424
5.2140084	.0188733774	.026533782	.0325614184	.952529207	54.575923
5.72236778	.0177264567	.0211973058	.0276324635	.874333383	50.0956306
6.28029158	.0162567436	.0167939828	.0233734801	.801651719	45.9312765
6.89261226	.0146344848	.0132103683	.0197150191	.734297995	42.0721911
7.56463344	.0129813711	.0103277974	.0165885321	.672043691	38.5052809
8.30217587	.0113794273	8.03199572E-03	.0139285434	.61463121	35.2157869
9.11162777	9.87985853E-03	6.21876925E-03	.0116741036	.561785109	32.1879273
10.0000002	8.51098595E-03	4.79675383E-03	9.76963303E-03	.5132213	29.405425

Figure 20

Figure 20 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** BODE PLOT (AMPLITUDE) **

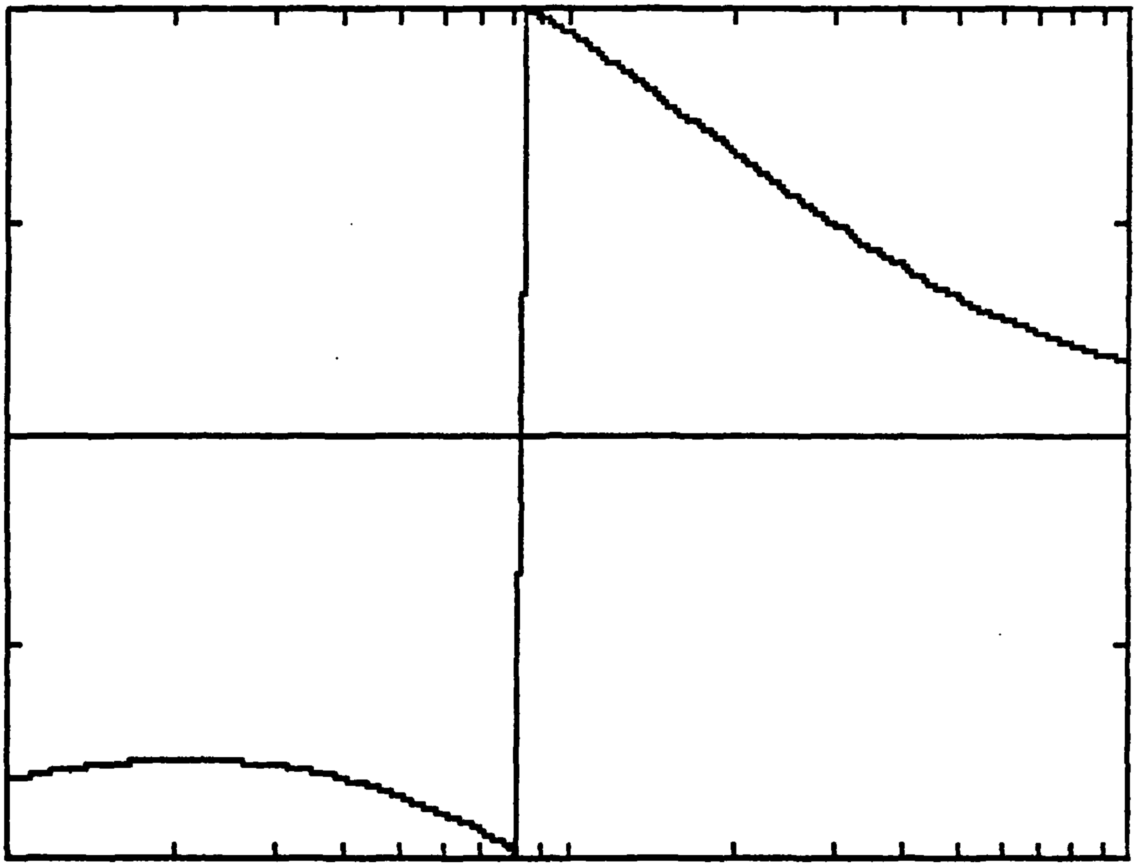


ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> COMMON LOG OF AMPLITUDE
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 10 RADIANS/SEC
AMPLITUDE LIMITS OF BODE PLOT ARE +-60 DECIBELS

Figure 20

Figure 20 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** BODE PLOT (PHASE) **



ABSCISSA -> COMMON LOG OF FREQUENCY
ORDINATE -> PHASE (DEGREES)
TIC MARKS SHOW MULTIPLES OF 90 DEGREES
MINIMUM FREQUENCY SHOWN ON ABSCISSA = .1 RADIANS/SEC
MAXIMUM FREQUENCY SHOWN ON ABSCISSA = 10 RADIANS/SEC
MAXIMUM PHASE ON ORDINATE SCALE = 180 DEGREES
MINIMUM PHASE ON ORDINATE SCALE = -180 DEGREES

Figure 20

ROOT LOCUS
PROBLEM IDENTIFICATION - KATZ EXAMPLE
****

NUMERATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

-.1
-.9
1

OPEN-LOOP ZEROS

REAL PART	IMAGINARY PART
-.1	0
1	0

DENOMINATOR COEFFICIENTS - IN ASCENDING POWERS OF S OR W'

0
0
4.6225
4.3
1

OPEN-LOOP POLES

REAL PART	IMAGINARY PART
-2.15	0
-2.15	0
0	0
0	0

MIN GAIN	MAX GAIN
0	-30

****

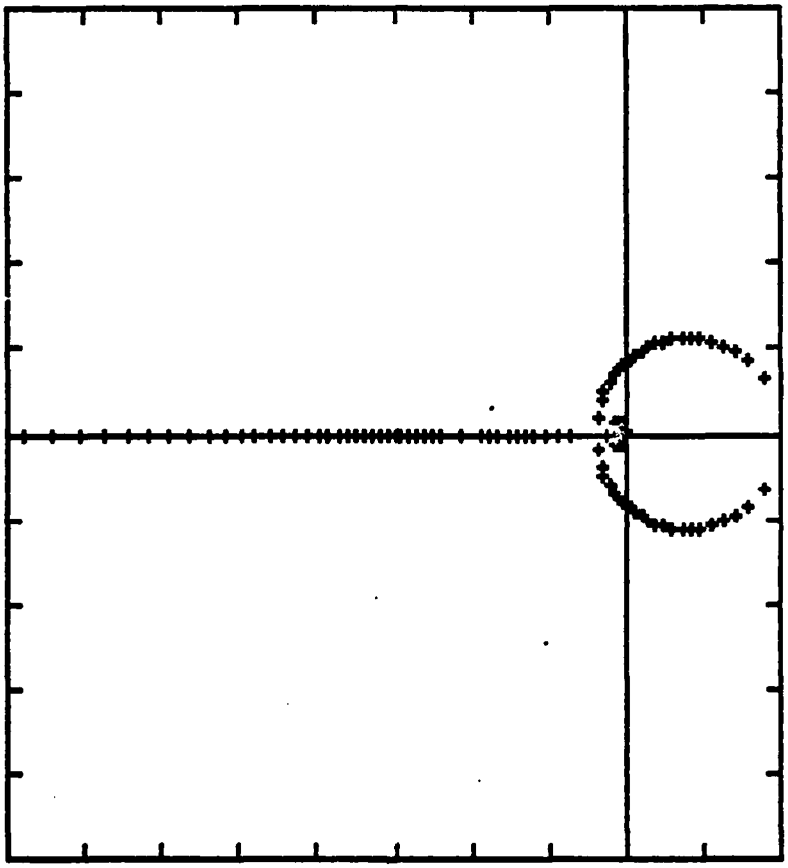
OPTION TAKEN

SIGMA MIN = -1 SIGMA MAX = 1
OMEGA MIN = -1 OMEGA MAX = 1

Figure 21 Compensated System Root Locus

Figure 21 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** ROOT LOCUS PLOT **

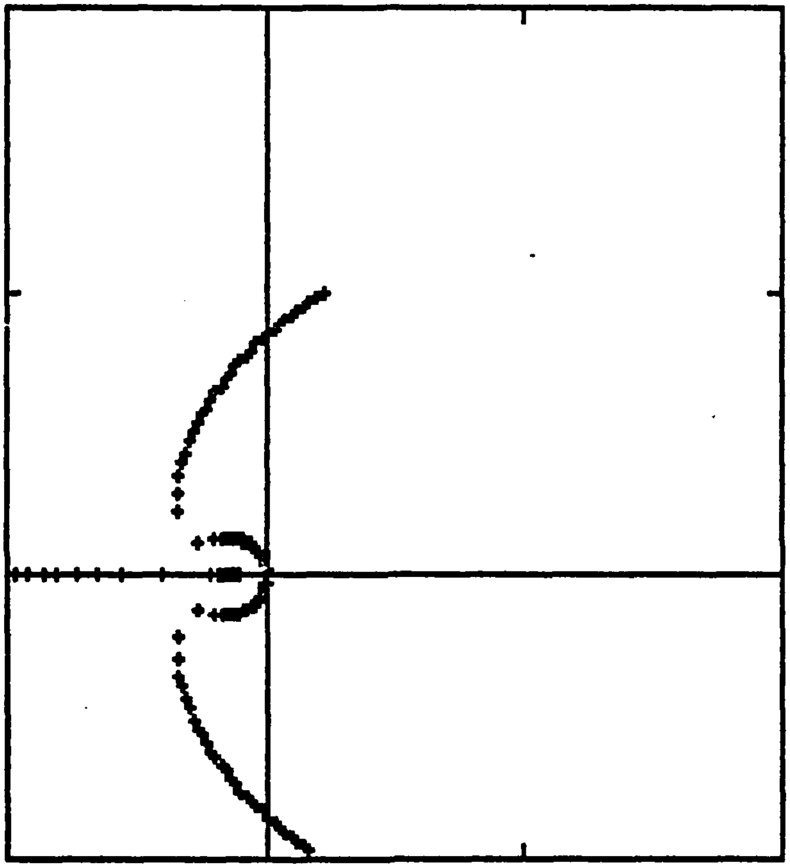


ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -8 TO 2
ORDINATE, -5 TO 5

Figure 21

Figure 21 (Cont.)

PROBLEM IDENTIFICATION - KATZ EXAMPLE
** ROOT LOCUS PLOT **



ABSCISSA -> REAL (SIGMA) AXIS
ORDINATE -> IMAGINARY (OMEGA) AXIS
TIC MARKS SHOW INTERVALS OF 1
THE PLOT FRAME LIMITS ARE:
ABSCISSA, -1 TO 2
ORDINATE, -1 TO 2

Figure 21

VI. CONCLUSIONS AND RECOMMENDATIONS

This thesis demonstrates that common classical control programs may be adapted to run on an inexpensive microcomputer system. With the exception of the memory management problem encountered, this was done with relative ease. This allows more people access to these programs by introducing a new group of computers on which they can be run. Also these programs require no knowledge of any computer language or input card formats making them easier to use than the Fortran versions.

It is also demonstrated that the transfer function programs are a useful tool in the study of sampled data systems as well as classical systems.

Only a sampling of five programs was converted in this thesis. Other programs can and should be similarly converted to run on microcomputer systems. There are programs in existence for the same microcomputer system used in this thesis that generate the aircraft stability derivatives. An effort should be made to modify these programs and the programs of this thesis to make them compatible and complementary.

APPENDIX A

DESCRIPTION OF MICROCOMPUTER SYSTEM

The microcomputer system used in developing this thesis consisted of the following components:

Apple II plus computer (48K)

Disk II 5 1/4" floppy disk drive and controller card

USI 9" green screen monitor

NEC PC-8023A-C dot matrix printer

Grappler printer interface card

Add Ram 16K expansion card

The programming language used was Applesoft basic. All graphs were generated using the High Resolution graphics commands. In the High Resolution graphics mode a matrix of dots 280 dots wide and 192 dots high can be displayed. The High Resolution page one is the only page used in this thesis and it resides in memory in the 8,192-byte area from $2000 to $3FFF. High Resolution page two resides in the area from $4000 to $6000.

A memory management problem was encountered during the programming of the programs requiring the use of the graphics capabilities of the microcomputer system. To understand the nature of the problem encountered a brief description of the normal use of memory in the Apple II is necessary. An Applesoft program is normally loaded at memory location $800 and loads up. LOMEM is set to the end of the program. Simple variables are stored from LOMEM upward as they are defined in the program. This gives only 6K-bytes of RAM before there is a conflict with the first High-Resolution page and only 14K before there is a conflict with the second High-Resolution page. [Ref. 8] Since the programs and variables in this thesis exceed 14K, simply using High-Resolution page two instead of one is not the answer. The first part of the solution was to get the Applesoft programs to load above the space used by High-Resolution page one and in effect protecting that space from interference. This fix creates a new problem. The disk operating system loads at the top of the 48K of memory and sets HIMEM to $9CF8. String variables start at HIMEM and build down. This allows 22K of space for the program and all variables. Some programs in this thesis require more space than this. Even with a 16K memory expansion card installed the disk operating system will ignore it so the added memory is useless. This final problem was solved by using a utility program that relocated the disk operating system into the higher memory provided by the memory expansion card and resets HIMEM to $BF00. This gives 32.5K of useable program space which is sufficient for all programs in this thesis. [Ref. 9]

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